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THE STUDENT'S BOOK OF SCALES

A. W. HOLOHAN.

THE

BOOK OF SCALES;

PRINCIPALLY DESIGNED FOR THE USE
OF
Students
PREPARING FOR ENTRANCE INTO
THE ROYAL MILITARY ACADEMIES
AT
Sandhurst and Woolwich.

BY

AMBROSE WHEELER HOLOHAN.

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PREFACE.

A knowledge of the construction and use of Scales, in the study of Fortification and Topography, is of paramount importance to Students entering the Military Academies at Sandhurst and Woolwich; and though many excellent treatises have been published on Geometrical Drawing, in each of which one or two chapters are devoted to Scales, the number of examples in these books are too few; hence the Student has a difficulty in getting a well-grounded knowledge of their construction. Almost invariably, when I asked a Student who had done something in Geometrical Drawing, but wanted further instruction, What was his weak point on the subject? I received the answer, "Scales." This led me to conclude that something more extensive—more explanatory—more illustrative, was necessary than what had already appeared on Scales; and in the present book I have endeavoured to supply the want.

A. W. H.

LONDON.

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TABLE OF LINEAL MEASURES.

The ordinary lineal measures are inches, links, feet, yards, fathoms, perches, chains, furlongs, and miles.

ENGLISH LINEAL MEASURES.

1 LEAGUE = 3 MILES.

Miles.	Furlongs.	Chains.	Poles or Perches.	Fathoms.	Yards.	Feet.	Links.	Inches.
1	8	80	320	880	1,760	5,280	8,000	63,360
	1	10	40	110	220	660	1,000	7,920
		1	4	11	22	66	100	792
			1	2¾	5½	16½	25	198
				1	2	6	9 1/11	72
					1	3	4 6/11	36
						1	1 17/33	12
							1	7·92

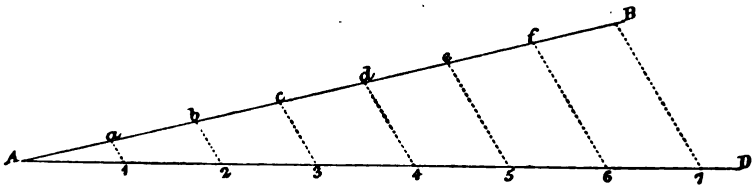
UNITS OF LINEAL MEASURE IN TERMS OF ENGLISH MEASURE.

Country.	Unit.	Feet.	Yards.	Miles.
Austria	Zoll (12 Linien)	·0864	·0288	—
„	Fuss or Schuh (12 Zoll)	1·0371	·34568	—
„	Elle	2·5586	·85289	—
„	Klafter (6 Fuss)	—	2·0741	—
„	Ruthe (10 Fuss)	—	3·4568	—
„	Meile (4,000 Klafter)	—	8,297·0	4·7142
„	Meile (Geographische)	8,101·0	4·6026	—
Belgium	Fuss (11 Zolle)	·90466	·30155	—
„	Elle	—	·74845	—
„	Verge	—	4·9255	—
„	Meile	—	4,860·833	2·7641
France	Millimètre	·0033	·0011	—
„	Centimètre	·0327	·0109	—
„	Décimètre	·3279	·1093	—
„	Mètre	—	1·0936	—
„	Kilomètre	—	1,093·63	·62138
„	Myriamètre	—	10,936·33	6·2138
Hanover	Fuss	·9579	·3193	—
Holland	Fuss	·9288	·3096	—
„	Meile	—	6,404·12	3·6387
Naples	Palma	·8628	·2876	—
„	Canna	—	2·3008	—
Portugal	Pie	1·08266	·3608	—
„	Palmo	·7171	·239	—
Prussia	Fuss (Rheinish Foot)	1·0297	·3432	—
„	Elle	—	·7293	—
„	Schritt	—	·8234	—
„	Klafter or Faden (6 Fuss)	—	2·0594	—
„	Ruthe (12 Fuss)	—	4·1188	—
„	Meile (2000 Ruthe)	—	·8272	4·7
Rome	Piede	·9665	·3222	—
Russia	Archine	—	·7777	—
„	Sachine (3 Archines)	—	2·3332	—
„	Verst (500 Sachines)	—	1,166·6	·6628
Saxony	Fuss	·9292	·3097	—
„	Meile (24,000 Fuss)	—	7,432·8	4·227
Spain	Pulgado	·077	·0257	—
„	Palmo (9 Pulgados)	·695	·2283	—
„	Pie, Castilian (12 Pul.)	·9273	·3091	—
„	Vara (3 Pies)	2·7682	·927	—
Sweden	Foot (10 Inches)	·9742	·3247	—
„	Alner	1·9484	·6494	—
„	Mile (18,000 Alners)	11,690·0	6·6423	—

TO DIVIDE A GIVEN FINITE STRAIGHT LINE INTO ANY NUMBER OF EQUAL PARTS.

In order to construct the different scales, the student will find it necessary to be able to divide a straight line into any number of equal parts. An example is here given.

Suppose it is required to divide a given line A B into 7 equal parts.



From A draw A D, making an angle with A B. Any angle less than a right angle will be a convenient one. Take any distance (suppose half an inch) between the points of the dividers, and step it off 7 times on A D. Join B 7, and through the points 6, 5, 4, &c., draw lines parallel to B 7, cutting A B in *a b c d e f*. A B is thus divided into 7 equal parts.

ON SCALES IN GENERAL.

A scale is a line with divisions marked upon it. It is used for measuring lines and angles, and for laying down distances.

Lines are measured by some determinate unit, as an inch, a foot, a yard, &c.

A degree is the unit of measurement for angles, &c.

A scale may be constructed on paper. Those sold by mathematical instrument makers are drawn on wood, ivory, metal, &c.

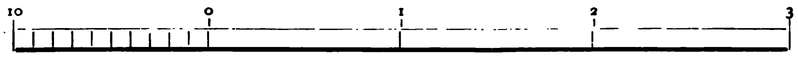
Scales are named according to the purposes for which they are used—such as the plain scales, scale of chords, plotting scale, &c.

A scale of equal parts is formed by laying off a number of equal divisions on a line, and subdividing one of them into any convenient number of equal parts.

For general purposes 10 is the ordinary number of equal parts into which one of the large divisions is subdivided. In particular plans the large division may be divided into as many parts as it represents feet, inches, &c.

In Fig. 1 the straight line A B is divided into 4 equal parts, and the left primary division is subdivided into 10 equal divisions. The line A B thus divided is a scale of equal parts.

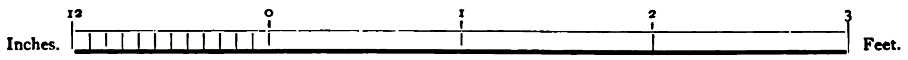
Fig. 1.



Lines of equal parts are of two sorts, viz., simply divided lines and diagonally divided lines.

The simply divided line is formed by drawing two or three parallel lines at about a tenth or twelfth of an inch apart, and of any convenient length. Divide this length into any number of equal parts as may be necessary. Each of these parts may represent an inch, a foot, a yard, &c. Fig. 2 represents a simply divided line. Each of the large or primary divisions represents a foot, and is subdivided into 12 equal parts, each of which represents an inch.

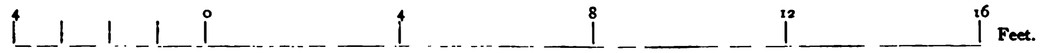
Fig. 2.



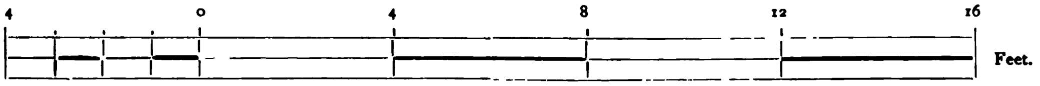
Each of the large divisions may represent a yard, a mile, &c., and its subdivisions would represent divisions of a yard, such as feet, &c. If the large division represent a mile, its subdivisions may represent furlongs, &c. In making the primary divisions, the divisional lines are drawn a little above the top line, say about half the space between the parallel lines.

It is quite a matter of taste whether a single line or two or three parallel lines are drawn in constructing a simply divided scale. Thus, for example, a scale representing 4 feet to an inch may be drawn in any of the following ways; each is equally correct:—

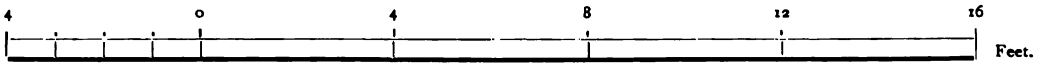
a.



b.



c.



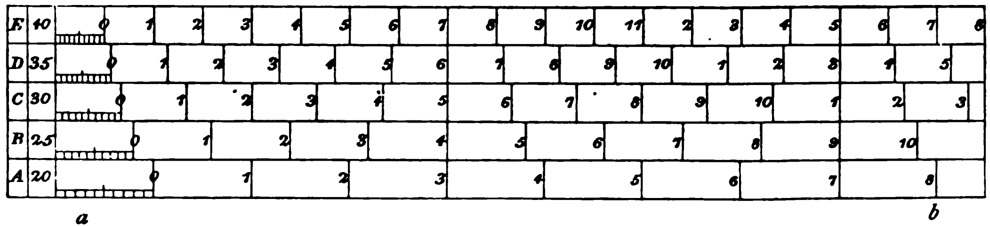
(*a*) Is the style of scale frequently marked on geographical maps.
(*b*) Is generally put on military surveys, &c.
(*c*) Is the form most used.

Heavy lines in a scale are merely for contrast, to add to the appearance of the scale.

There are usually several of these simply divided scales put on rulers, one above the other, as in Fig. 3.

Fig. 3.

SIMPLY DIVIDED DECIMAL SCALES.

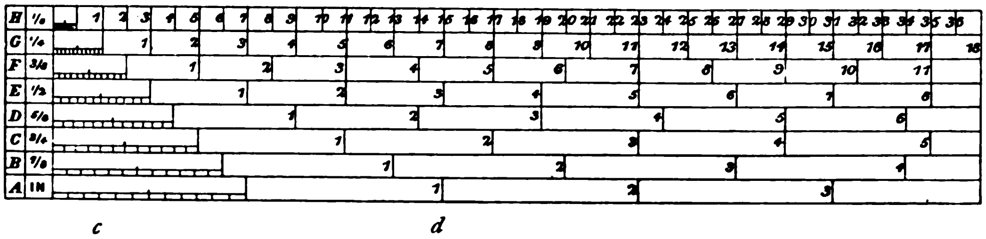


In each scale the numbers on the left hand show into how many equal parts each inch is divided, such as 20, 25, 30, 35, 40, &c. Thus in Scale A the inch is divided into 20 parts, and half-an-inch is represented on the scale divided into 10 parts. In Scale B the inch is divided into 25 parts, and two-fifths of the inch is divided into 10 parts. In Scale C the inch is divided into 30 parts, and two-sixths, or one-third, of the inch is divided into 10 parts. In Scale D the inch is divided into 35 parts, and two-sevenths of the inch in divided into 10 parts. In Scale E the inch is divided into 40 parts, and the two-eighths, or one-fourth, of the inch is divided into 10 parts. These scales are called decimal scales, and are used as the plan to be represented may require.

Another set of plain scales, duodecimally divided, are also used. Fig. 4 represents a set of duodecimal scales.

Fig. 4.

SIMPLY DIVIDED DUODECIMAL SCALES.



NOTE.—Sometimes the decimal scales are marked on the same side of the ruler with the duodecimal.

The parallel lines are ruled for these scales as in the decimal scales, Fig. 3, but the inch on the Scale A is divided into 12 equal parts. If the inch be taken to represent a foot, each twelfth part of it represents one inch. In Scale B seven-eighths of an inch represents a foot, and is divided into 12 equal parts, each of which represents an inch. In Scale C three-fourths of an inch represents a foot, and so on to the top Scale, H, in which one-eighth of an inch represents a foot.

The use of these Scales of equal parts is to lay down any line expressed by any number of two places, or denominations, whether decimally or duodecimally divided as miles, yards, feet, &c., and their tenth or twelfth parts; thus, if each division in the decimal scale be taken as 1 mile, 1 yard, &c., then each of the subdivisions will express 1-10th part thereof; and if each of the large divisions be called 10, then each subdivision will be 1; and if the large divisions be 100, then each subdivision will be 10; and so on.

Now suppose you want to lay down from Scale A, Fig. 3, a line to represent $8\frac{7}{10}$ths, 87, or 870 miles, yards, feet, &c., place one point of the dividers on the eight of the large divisions, counting from the left hand towards the right, and extend the other point to the seventh of the small divisions, counting from the right hand towards the left; then the space between the points of the dividers will represent a line $8\frac{7}{10}$ths, 87, or 870 miles, yards, &c., and it bears such a proportion to the plan as the line measured does to the thing represented. This distance is shown on the scale by the letters *a, b,* Fig. 3.

If a length of feet and inches were to be laid down from the duodecimal scales, Fig. 4.

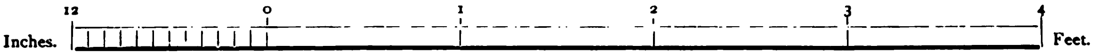
The large divisions may represent the feet, but the inches must be taken from the part of the first division which is divided into 12 equal parts.

Suppose it is required to take off a line from Scale A, Fig. 4, representing 1 foot 9 inches. Set one point of the dividers on the first large division, counting from the left hand to the right, and extend the other point of the dividers to the ninth small division, counting from the right to the left. The distance between the points of the dividers will represent 1 foot 9 inches. This distance is shown on the Scale A by the letters *c*, *d.*

EXAMPLE 1.

Construct a plain scale of feet and inches, when one inch represents a foot. Show 5 feet.

Fig. 5.



Draw with the lead pencil two parallel lines of any convenient length, and about 1-10th of an inch apart. In inking the lines, make the lower one a little heavier than the upper line. Divide the lower line into inches, as 1, 2, 3, 4, &c.; each inch represents a foot. Subdivide the left primary division into 12 equal parts; each subdivision represents one inch.

Suppose you want to lay down, with this scale, a line 3 feet 3 inches long.

Place one leg of the dividers on the division marked 3, counting from the left to the right, and extend the dividers to the third subdivision, counting from right to left; the space between the points of the dividers represents 3 feet 3 inches.

Set down a line representing 5 feet 7 inches.

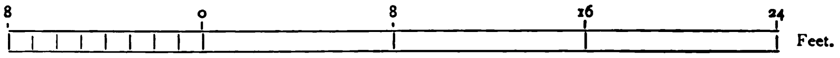
Draw a rectangle 3 feet 6 inches long, and 2 feet 3 inches broad.

Describe a circle with a radius of 1 foot 5 inches.

EXAMPLE 2.

Construct a scale 8 feet to an inch.

Fig. 6.



Draw the parallel lines, as in the last example, of any convenient length, and divide the lowest into inches. Subdivide the left primary division into 8 equal parts; each of these parts represent a foot.

Suppose you want to lay down with this scale two lines, respectively 24 feet and 32 feet long.

Place one point of the dividers at 0 and extend the other point to 24. The space between the points of the dividers represents 24 feet. Again place one point of the dividers at the subdivision marked 8, and extend the other point to 24; the space between the points represents 32 feet.

1. Describe a circle with a radius of 7 feet.
2. Construct a rectangle, the sides of which are respectively 17 feet and 21 feet.
3. The side of a square is 14 feet, what is the length of the diagonal?

EXAMPLE 3.

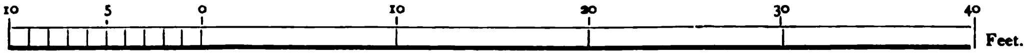
Construct a scale to measure 50 feet, when one inch represents 10 feet.

First find what length of line will be required to represent 50 feet.

Say as 10 feet : 50 feet : : 1 foot : x inches = 5 inches.

Fig. 7.

SCALE 10 FEET TO AN INCH, OR $\frac{1}{120}$.



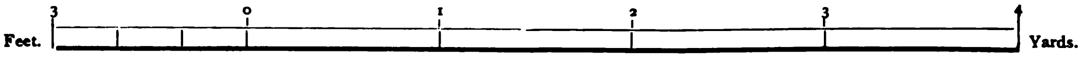
Draw the parallel lines each 5 inches long, and the same distance apart as in the former examples. Divide the lowest line into 5 equal parts; each of these parts represents 10 feet. Subdivide the left primary division into 10 equal parts; each subdivision represents one foot. Complete the scale and number it as in the example.

EXAMPLE 4.

Construct a scale one yard to an inch, to show feet.

Fig. 8.

SCALE 1 YARD TO AN INCH, OR $\frac{1}{36}$.



NOTE.—One inch in a scale may represent a foot, a yard, a mile, &c. In such cases the scale is said to be "one foot to an inch," "one yard to an inch," "one mile to an inch," &c.

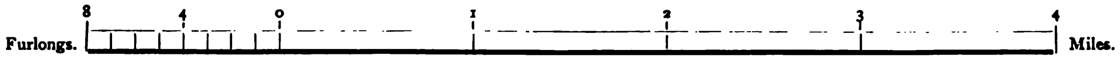
Draw the parallel lines any convenient length, 4, 5, or 6 inches long. Divide the lowest line into inches; each inch represents 1 yard. Subdivide the left primary division into 3 equal parts; each subdivision represents 1 foot. Number the scale as in the example.

EXAMPLE 5.

Construct a scale one mile to an inch, to show 40 furlongs.

Fig. 9.

SCALE 1 MILE TO AN INCH, OR $\frac{1}{63,360}$.



There are 8 furlongs in a mile: then 40 divided by 8 goes 5 times. Therefore the scale will be 5 inches long. Draw the parallel lines 5 inches long. Divide the lowest line into inches, each inch represents 1 mile. Subdivide the left primary division into 8 equal parts; each subdivision represents one furlong. Number the scale as in the example.

EXAMPLE 6.

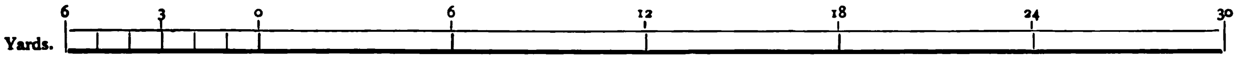
Construct a scale to measure 36 yards, when 1 inch represents 6 yards.

$$\overset{\text{yds.}}{6} : \overset{\text{yds.}}{36} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 6 \text{ inches.}$$

As 6 : 36 :: 1 : x = 6 inches.

Fig. 10.

SCALE 6 YDS. TO AN INCH, OR $\frac{1}{216}$.



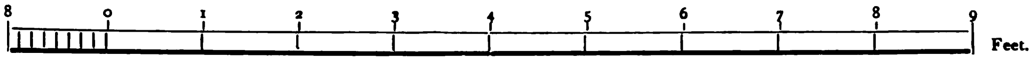
Draw the parallel lines 6 inches long. Divide the lowest into inches; each inch represents 6 yards. Subdivide the left primary division into 6 equal parts; each subdivision represents 1 yard. Number the scale as in the example.

EXAMPLE 7.

Construct a scale 2 feet to an inch, to show 8ths of a foot.

Fig. 11.

SCALE 2 FEET TO AN INCH, OR $\frac{1}{24}$.



Assume any convenient length, as 5 inches, divide it into 10 equal parts, each part represents 1 foot. Subdivide the left primary division into 8 equal parts; each subdivision represents $\frac{1}{8}$th of a foot. Complete the scale as in the example.

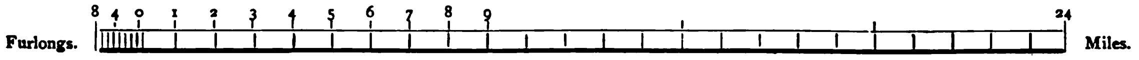
EXAMPLE 8.

The distance between two points on a map is 5 miles, and is represented by 1 inch. Construct a scale to measure 25 miles, and show furlongs.

$$\overset{\text{miles.}}{5} : \overset{\text{miles.}}{25} :: \overset{\text{in.}}{1} : x = 5 \text{ inches.}$$

Fig. 12.

SCALE 5 MILES TO AN INCH, OR $\frac{1}{316,800}$.



Draw the parallel lines 5 inches long, and divide the lowest into inches; each inch represents 5 miles. Subdivide each inch into 5 equal parts; each part represents 1 mile. Again divide the left primary subdivision into 8 equal parts; each of these latter parts represents 1 furlong. Number the scale as in the example.

ON THE REPRESENTATIVE FRACTION.

The representative fraction is simply the proportion between any line on the drawing and the corresponding line on the original object. Suppose the fraction $\frac{1}{12}$th marked on a drawing. This means that each line on the drawing is $\frac{1}{12}$th the length of a corresponding line on the building or other original object. For instance, if the width of a door on the plan or drawing is represented by 4 inches, these 4 inches represent $\frac{1}{12}$th of the actual door, consequently the size of the door is $4 \times 12 = 48$ inches or 4 feet.

Again, suppose the length of a building is 100 feet, and is represented on a scale by 5 inches; the proportion then of the drawing to the original is as 5 inches to 100 feet, or 1 inch to 20 feet; but in 20 feet there are $20 \times 12 = 240$ inches; therefore the length of the building on the drawing is 240 times smaller than the actual building, and the representative fraction is consequently $\frac{1}{240}$.

A line, marked $\frac{1}{48}$th, shows that each line on the plan is 48 times less than the corresponding line on the original object; and so on.

EXAMPLES.

1. A scale is 1 foot to an inch, what is the representative fraction?

Find the number of inches which 1 inch on the scale represents, and that will give you the representative fraction.

Here 1 inch represents 12 inches, therefore the representative fraction is $\frac{1}{12}$.

2. A scale is 8 feet to an inch, find the representative fraction.
Here 1 inch represents 8 feet, and as in 8 feet there are 8 × 12 = 96 inches; therefore the representative fraction is $\frac{1}{96}$.

3. Find the representative fraction of a scale to measure 50 feet, when 1 inch represents 10 feet.
Here 10 feet multiplied by 12 = 120; the representative fraction is $\frac{1}{120}$.

4. In a scale 1 yard to an inch, find the representative fraction.
One yard = 3 feet, multiplied by 12, gives 36 inches.
One inch then represents 36 inches; the representative fraction therefore is $\frac{1}{36}$.

5. In a scale 1 mile to an inch, find the representative fraction.
Here 1 inch represents 1 mile = 5,280 feet = 63,360 inches; the representative fraction is $\frac{1}{63,360}$.

6. In a scale 6 yards to 1 inch, find the representative fraction.
Here 6 yards = 18 feet = 216 inches; the representative fraction is $\frac{1}{216}$.

7. In a scale of 2 feet to an inch, find the representative fraction.
Here 1 inch represents 2 feet = 24 inches; the representative fraction is $\frac{1}{24}$.

8. The distance between two places on a map is 5 miles, and is represented by 1 inch, find the representative fraction.
Here 5 miles, multiplied by 5,280, the number of feet in a mile = 26,400, which multiplied by 12, the number of inches in a foot, gives 316,800 inches, and the representative fraction is $\frac{1}{316,800}$.

The representative fraction may also be found by making the distance represented the *denominator* of a fraction, and the number which represents that distance the *numerator* of the fraction; and then, if necessary, reducing the fraction to its lowest terms. This is simply another method of finding the number of inches represented by 1 inch.

9. Suppose 7 feet are represented by 3 inches, what is the representative fraction?

The number which represents the distance	3 inches
The distance represented	7 feet

Bring the feet to inches, and reduce the fraction.

$$\frac{3\text{ in.}}{7\text{ ft.}} = \frac{3}{7 \times 12} = \frac{3}{84} = \frac{1}{28}.$$ The representative fraction.

10. If 6 miles are represented by 4 inches, what is the representative fraction?

$$\frac{4\text{ in.}}{6\text{ miles}} = \frac{4}{6 \times 1760 \times 36} = \frac{4}{380,160} = \frac{1}{9,504}.$$ The representative fraction.

TO CONSTRUCT THE SCALE, THE REPRESENTATIVE FRACTION BEING GIVEN.

EXAMPLE 9.

The representative fraction is $\frac{1}{240}$. Construct a scale to measure 100 feet.

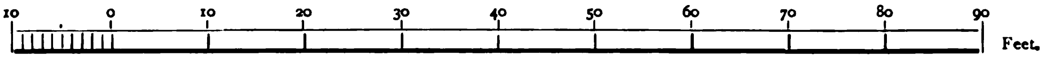
Here 1 inch is the 240th part of the original; consequently 1 inch represents 240 inches, or 20 feet.

$$\text{As } \overset{\text{ft.}}{20} : \overset{\text{ft.}}{100} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 5 \text{ inches.}$$

Set down a line 5 inches long. This line represents 100 feet. Divide it into 10 equal parts, each part represents 10 feet. Subdivide the left primary division into 10 equal parts; each subdivision represents 1 foot. Number the scale as in the example.

Fig. 13.

SCALE 20 FEET TO AN INCH OR $\frac{1}{240}$.



EXAMPLE 10.

The representative fraction is $\frac{1}{4800}$. Construct a scale to measure from 10 to 600 yards.

Here 1 inch is $\frac{1}{4800}$th part of the original; therefore 1 inch represents 4,800 inches.

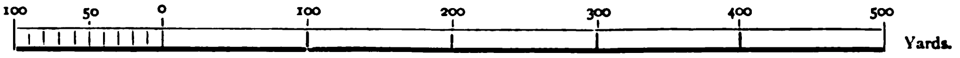
4,800 ÷ 12 = 400 feet, which, divided by 3, gives 133·3 yards.

To find the length of the scale, say—

$$\text{As } \overset{\text{yds.}}{133{\cdot}3} : \overset{\text{yds.}}{600} :: \overset{\text{in.}}{1} : x = 4{\cdot}5 \text{ inches.}$$

Fig. 14.

SCALE $\frac{1}{4800}$.



Set down a line 4·5 inches long. This line represents 600 yards. Divide it into 6 equal parts; each part represents 100 yards.

Subdivide the left primary division into 10 equal parts; each subdivision represents 10 yards. Complete the scale as in the example.

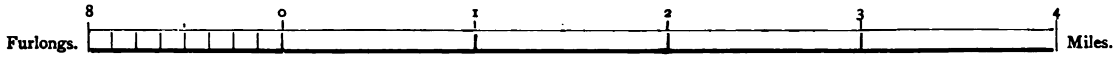
EXAMPLE 11.

The representative fraction is $\frac{1}{63,360}$. Construct a scale to measure 5 miles, and show furlongs.

Here 1 inch represents 63,360 inches, which divided by 12 = 5,280 feet, or 1 mile. The scale is then 1 mile to an inch; therefore a line 5 inches long will represent 5 miles.

Fig. 15.

SCALE $\frac{1}{63,360}$.



Divide a line 5 inches long into 5 equal parts; each part represents 1 mile. Subdivide the left primary division into 8 equal parts; each subdivision represents 1 furlong. Complete the scale as in the example.

EXAMPLE 12.

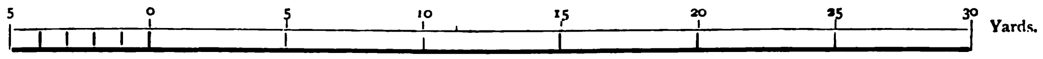
The representative fraction is $\frac{1}{252}$. Construct a scale to show single yards and measure 35 yards.

Here 1 inch represents 252 inches, which divided by 36, the number of inches in a yard, = 7 yards.

$$\text{As } \overset{\text{yds.}}{7} : \overset{\text{yds.}}{35} :: \overset{\text{in.}}{1} : x = 5 \text{ inches.}$$

Fig. 16.

SCALE $\frac{1}{252}$.



Draw a line 5 inches long, and divide it into 7 equal parts; each division represents 5 yards. Subdivide the left primary division into 5 equal parts; each subdivision represents 1 yard. Number the scale as in the example.

EXAMPLE 13.

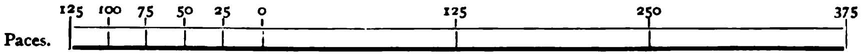
The representative fraction is $\frac{1}{3750}$. Construct a scale of 1,000 paces, to show 25 paces of 30 inches each.

Here 1 inch represents 3,750 inches, which divided by 30, the number of inches in 1 pace, gives 125 paces. One inch then represents 125 paces.

$$\text{As}\ \overset{\text{paces.}}{125} : \overset{\text{paces.}}{1{,}000} :: \overset{\text{in.}}{1} : x = 8\ \text{inches.}$$

Fig. 17.

SCALE $\frac{1}{3750}$.



Draw a line 8 inches long, and divide it into 8 equal parts; each division represents 125 paces. Subdivide the left primary division into 5 equal parts; each subdivision represents 25 paces. Half the scale is represented in Fig. 13.

EXAMPLE 14.

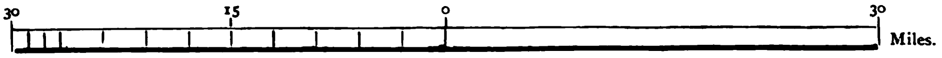
The representative fraction is $\frac{1}{844{,}800}$. Construct a scale to show 60 miles.

Here 1 inch represents 844,800 inches. 844,800 ÷ 12 = 70,400 feet, which divided by 5,280, the number of feet in a mile, gives $13\frac{1}{3}$ miles. One inch, then, represents $13\frac{1}{3}$ miles.

$$\text{As}\ \overset{\text{mi.}}{13\tfrac{1}{3}} : \overset{\text{mi.}}{60} :: \overset{\text{in.}}{1} : x = 4{\cdot}5\ \text{inches.}$$

Fig. 18.

SCALE $\frac{1}{844{,}800}$.



Draw a line 4·5 inches long and divide it into 2 equal parts, each part represents 30 miles. Subdivide the left division into 10 equal parts; each subdivision represents 3 miles. Divide the left primary subdivision into 3 equal parts, each of which represents 1 mile.

Complete the scale as in the example.

EXAMPLE 15.

The representative fraction is $\frac{1}{168,960}$.

Construct a scale to measure 14 miles and show furlongs.

Here 1 inch represents 168,960 inches.

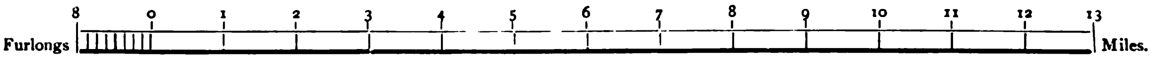
168,960 ÷ 12 = 14,080 feet, which divided by 5,280, the number of feet in a mile, gives $2\frac{2}{3}$ miles.

One inch then represents $2\frac{2}{3}$ miles.

$$\text{As } \overset{\text{mi.}}{2\tfrac{2}{3}} : \overset{\text{mi.}}{14} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 5{\cdot}25 \text{ inches.}$$

Fig. 19.

SCALE $\frac{1}{168,960}$.



Draw a line 5·25 inches long, and divide it into 14 equal parts, each of which represents 1 mile. Subdivide the left primary division into 8 equal parts; each subdivision represents 1 furlong. Number the scale as in Fig. 19.

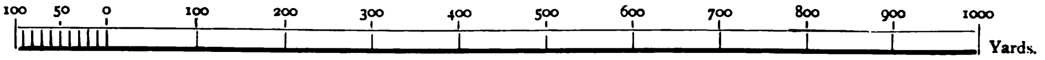
EXAMPLE 16.

The representative fraction is $\frac{1}{7920}$. Construct a scale to measure 1,100 yards and show 10 yards.

Here 1 inch represents 7,920 inches, which divided by 12 gives 660 feet, and this divided by 3 gives 220 yards. One inch then represents 220 yards.

$$\text{As } \overset{\text{yds.}}{220} : \overset{\text{yds.}}{1,100} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 5 \text{ inches.}$$

Fig 20.



Draw a line 5 inches long, and divide it into 11 equal parts; each division represents 100 yards. Subdivide the left hand primary division into 10 parts; each subdivision represents 10 yards.

EXAMPLE 17.

The representative fraction is $\frac{1}{10,560}$.

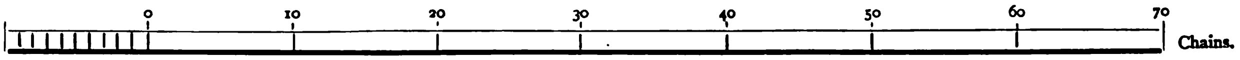
Construct a scale of chains to show 1 mile. The smallest unit 1 chain.

Here 1 inch represents 10,560 inches, or 880 feet, which is the $\frac{1}{6}$th of a mile. There are 80 chains in a mile. Then 80 divided by 6 gives $13\frac{1}{3}$ chains; therefore, 1 inch represents $13\frac{1}{3}$ chains.

$$\text{As } \overset{\text{chs.}}{13\tfrac{1}{3}} : \overset{\text{chs.}}{80} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 6 \text{ inches nearly.}$$

Fig. 21.

SCALE $\frac{1}{10,560}$.



Draw a line 6 inches long, and divide it into 8 equal parts; each division represents 10 chains. Subdivide the left primary division into 10 equal parts; each part represents 1 chain.

EXAMPLE 18.

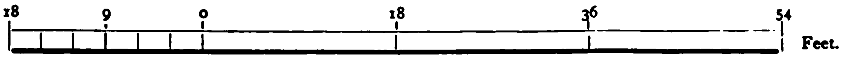
The distance between any two points on a map is $\frac{1}{216}$ of the real distance.

Construct a scale to measure feet.

Here 1 inch represents 216 inches, or 18 feet.

Fig. 22.

SCALE $\frac{1}{216}$.



Draw a line any convenient length, suppose 4 inches. Divide it into 4 equal parts; each division represents 18 feet. Subdivide the left primary division into 6 equal parts; each subdivision represents 3 feet.

COMPARATIVE SCALES.

When the same distances can be measured by two scales which are differently divided, but whose representative fraction is the same, then one of the scales is said to be comparative to the other. Thus the divisions on one scale may show miles, and the divisions on the other may show yards. This is sometimes expressed by saying, the unit of measure in one scale is a mile, and the unit of measure in the other is a yard. In making one scale comparative to another there must always be two units of measure.

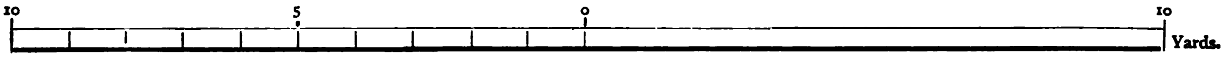
EXAMPLE 19.

Suppose it is required to construct a scale of yards comparative to (Ex. 3), in which 1 inch represents 10 feet. Any number of yards may be shown—suppose 20.

In 20 yards there are 20 × 3 = 60 feet.

$$\text{Then as } \overset{\text{ft.}}{10} : \overset{\text{ft.}}{60} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 6 \text{ inches.}$$

Fig. 23.



Draw a line 6 inches long. This line represents 60 feet, or 60 ÷ 3 = 20 yards.

Divide it into 2 equal parts; each division represents 10 yds. Subdivide the first primary division into 10 equal parts; each subdivision represents 1 yard.

The representative fraction in the scale (Ex. 3), 10 feet to an inch, is $\frac{1}{120}$, and the representative fraction in the comparative scale, 20 yards to 6 inches, is $\frac{6}{20 \times 3 \times 12} = \frac{6}{720} = \frac{1}{120}$. Thus the one scale is comparative to the other.

EXAMPLE 20.

In Fig. 5 the scale is 1 inch to a foot. Construct a comparative scale of French mètres; show décimètres and centimètres.

The mètre is the name of the fundamental unit which is the basis of the mètre system. Its equivalent in English feet is set down as 3 feet 3 inches and $\frac{3}{8}$ths of an inch, or 39·37079 inches. The mètre divided into 10 equal parts shows décimètres. A décimètre divided into

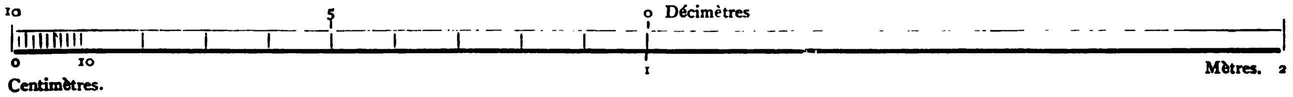
10 equal parts shows centimètres, and the centimètre divided into 10 equal parts shows millimètres.

$$\text{Thus 1 mètre} = \begin{cases} 10 \text{ décimètres.} \\ 100 \text{ centimètres.} \\ 1{,}000 \text{ millimètres.} \end{cases}$$

Then, to make a scale of French mètres comparative to any English scale, you have merely to find such a measurement as shall bear the same ratio to 39·37 inches that the English scale bears to the actual size. Thus, if 1 inch represents 12 inches, how many inches will represent 39·27 inches?

$$\text{As } \overset{\text{in.}}{12} : \overset{\text{in.}}{39{\cdot}27} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 3{\cdot}28 \text{ inches.}$$

Fig. 24.



Draw a line of any convenient length, and take between the points of the dividers from the diagonal scale 3·28 inches. Step off this distance (suppose twice) on the straight line; each of these divisions represents a mètre. Subdivide the left primary division into 10 equal parts; each part represents 1 décimètre; and 1 of these décimètres divided into 10 equal parts represents centimètres.

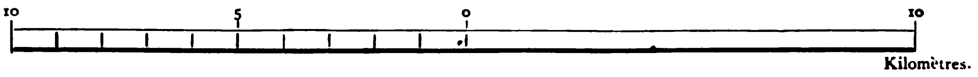
EXAMPLE 21.

An English scale showing 30 miles is represented on a map by 11·25 inches. Construct a comparative scale of 20 French kilomètres.

The kilomètre is the mile, or principal itinerary measure, of the mètre system, and is equivalent to 1,093·63 English yards. In 30 miles there are 30 × 1,760 = 52,800 yards. In 20 kilomètres there are 20 × 1,093·63 = 21,872·60 yards.

$$\text{Then as } \overset{\text{yds.}}{52{,}800} : \overset{\text{yds.}}{21{,}872{\cdot}60} :: \overset{\text{in.}}{11{\cdot}25} : \overset{\text{in.}}{x} = 4{\cdot}66 \text{ inches nearly.}$$

Fig. 25.



Take a line 4·66 inches long; these 4·66 inches represent 21,872·60 yards, or 20 kilomètres. Divide the line 4·66 inches into 2 equal parts; each division represents 10 kilomètres. Subdivide the left primary division into 10 equal parts; each subdivision represents 1 kilomètre.

EXAMPLE 22.

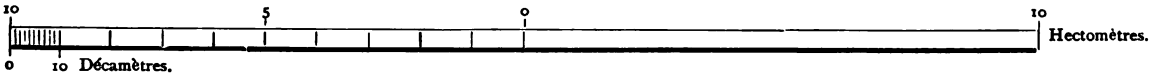
The scale on an English map is 4·25 inches to a mile. Construct a scale of French hectomètres and décamètres.

In 1 kilomètre there are 10 hectomètres.
„ „ „ 1,093·63 yards.
„ English mile „ 1,760 yards.

Then if 1,760 yards are represented by 4·25 inches, how many inches will represent 1,093·63 yards?

yds. yds. in. in.
As 1,760 : 1,093·63 : : 4·25 : x = 2·64 inches.

Fig. 26.



Draw a line any convenient length, and make on it spaces each equal to 2·64 inches. These spaces represent kilomètres. Divide one of them into 10 equal parts; each of these parts represents 1 hectomètre. Subdivide 1 hectomètre into 10 equal parts; each subdivision of the hectomètre represents 1 décamètre.

Number the scale as in the Example.

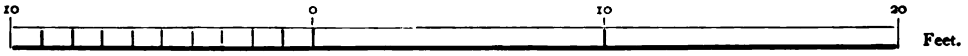
EXAMPLE 23.

The scale on a French plan is 20 décimètres to an inch. Construct an English scale to measure 30 feet.

A décimètre equals ·327 of an English foot, therefore 20 décimètres equal 20 × ·327 = 6·540 English feet, and are represented on the French plan by 1 inch. How many inches will represent 30 feet?

ft. ft. in. in.
As 6.54 : 30 : : 1 : x = 4·58 inches.

Fig. 27.



Draw a line 4·58 inches long, and divide it into 3 equal parts; each part represents 10 feet. Subdivide the left primary division into 10 equal parts; each subdivision represents 1 foot.

Number the scale as in Fig. 27.

EXAMPLE 24.

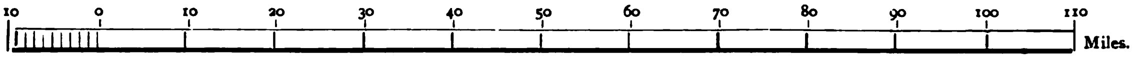
On a French map 20 leagues are represented by 2·25 inches. Construct a comparative scale showing 120 English miles.

The French league is equal to 4,262·84 English yards; therefore 20 leagues = 20 × 4,262·84 = 85,256·80 yards.

One English mile = 1,760 yards, therefore 120 miles equal 120 × 1,760 = 211,200 yards.

yds. yds. in. in.
As 85,256·80 : 211,200 : : 2·25 : x = 5·57 inches.

Fig. 28.



Divide a line 5·57 inches long into 12 equal parts; each part represents 10 miles. Subdivide the left primary division into 10 equal parts; each subdivision represents 1 mile.

Number scale as in Fig. 28.

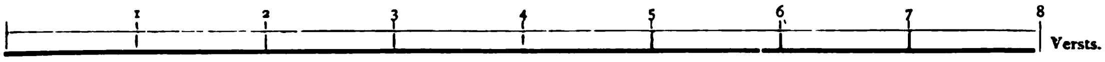
EXAMPLE 25.

The scale is 1 mile to an inch. Construct a comparative scale showing 8 Russian versts.

One Russian verst = 1,166·6 yards, therefore 8 Russian versts equal 8 × 1,166·6 = 9,332·8 yards.

yds. yds. in. in.
As 1,760 : 9,332·8 : : 1 : x = 5·30 inches.

Fig. 29.



Divide a line 5·30 inches long into 10 equal parts; each part represents 1 Russian verst. Complete the scale as in Fig. 29.

EXAMPLE 26.

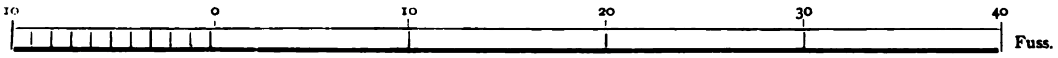
The representative fraction of an English scale is $\frac{1}{120}$. Construct a comparative scale of Prussian fuss. Show 50 fuss.

$\frac{1}{120}$ shows the English scale is 120 inches, or 10 feet, to an inch.

1 Prussian fuss equals 1·0297 feet, and 50 fuss equal 1·0297 × 50 = 51·4850 feet.

$$\text{Then as } \overset{\text{ft.}}{10} : \overset{\text{ft.}}{51{\cdot}4850} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 5{\cdot}14.$$

Fig. 30.



Draw a line 5·14 inches long; this represents 50 fuss. Divide it into 5 equal parts; each division represents 10 fuss. Subdivide the first primary division into 10 equal parts; each subdivision represents 1 fuss. Complete the scale as in Fig. 30.

Similarly, by reference to the table showing the units of linear measure in terms of English measure, any variety of comparative scales may be constructed.

DIAGONAL SCALES.

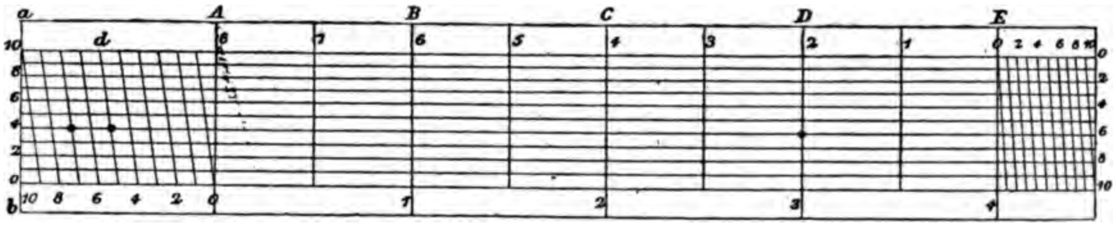
The use of the diagonal scale is much the same as the simple scale; but with the diagonal scale very minute distances can be measured with much greater accuracy.

From the construction of the scale (see Fig. 31) it is plain that, if each of the primary divisions represents 1, each of the 1st subdivisions represents $\frac{1}{10}$th of 1, and each of the 2nd subdivisions (taken on the diagonal lines, counting from the bottom upwards, or from the top downwards, according to the way on which the diagonals are drawn) will express $\frac{1}{10}$th of the former subdivisions, or a hundredth of the primary divisions. And if each of the primary divisions express 10, then each of the 1st subdivisions will express 1, and each of the 2nd

$\frac{1}{10}$th; and if each of the primary divisions represent 100, then each of the 1st subdivisions will be 10, and each of the 2nd will be 1. Consequently, to lay down a line whose length is 347, 34$\frac{7}{10}$, or 3$\frac{47}{100}$ miles, yards, &c., on the diagonal line joined to the 4th of the 1st subdivisions count 7 upwards, reckoning the distance of each parallel 1, and on this point place one point of the dividers, and extend the other to the intersection of the third primary division with the same parallel in which the other point rests, and the distance between the dividers will represent 347, 34$\frac{7}{10}$, 3$\frac{47}{100}$, &c.

Fig. 31.

DIAGONAL SCALE, SHOWING THE INCH AND HALF-INCH DIVIDED DIAGONALLY.



To construct the diagonal scale, draw 11 parallel lines about the same distance apart as in Fig. 19, and 5·5 inches long. The distance apart of the parallel lines is immaterial. Above and below the 11 lines draw 2 lines, *a*, *b*, about twice the distance apart. These double spaces are for numbering the scale. Divide the 11th parallel line into inches, having half-an-inch at the end to the right, and from the extremities of this line and the intermediate points of division erect perpendiculars 0, 1, 2, 3, 4, &c. Subdivide the left primary division into 10 equal parts. Join the first point of division, marked 9 on the lower line, with the angular point 10 on the left of the top line, and through the other points of subdivision draw lines parallel to this line. These lines are called diagonals. Number the scale as in Fig. 31, and it is completed. The half-inch at the end of the scale is also divided by diagonals, and this scale of half-inches is numbered on the top. Fig. 31 represents such a scale as is generally found on the flat side of the protractor. On some scales the half-inch and quarter-inch are divided diagonally.

Now I shall further explain the use of this scale. The primary division on the scale is subdivided into 10 equal parts, and by means of the parallel lines a tenth of each small subdivision may be taken. Units may be taken on the lowest line, and tenths of those units on the parallel lines. The inch in Fig. 31 being divided into 10 equal parts, then from 3 to 0, or from D to A, would be 30. From 3 to 5, or from D to *d*, would be 35; from 3 to 7 would be 37, &c.; but if the dividers be extended from the same vertical lines on the 4th parallel line, then the spaces would be 354, 374. If the inch were only one, those same spaces would be 3·54 and 3·74, and so on. The values of the spaces would vary according to the value of the inch. These spaces are shown on the scales by dots.

If the inch be divided into 10 equal parts and you want to show 10ths of these divisions, draw parallel lines enclosing 10 equal spaces. If the inch be divided into 12 equal parts and you want to show 12ths of these divisions, draw parallel lines enclosing 12 equal spaces. If the inch be divided into 10 equal parts, and that each of these parts represents a mile, and you want to show 8ths of these divisions, or furlongs, draw parallel lines enclosing 8 equal spaces. In a word, if you want to show any fractional part of the divisions of the inch, half-inch, or quarter-inch, draw equally distant parallel lines enclosing so many spaces; then, by drawing the diagonals, you get the fractional parts required.

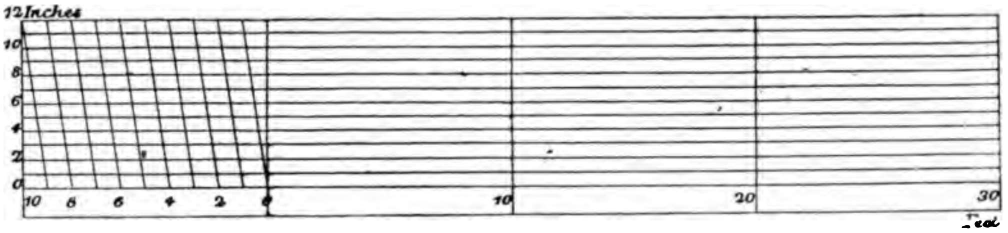
EXAMPLE 27.

Construct a diagonal scale 8 feet to an inch, to measure 40 feet, and show inches.
Find the required length of scale, as in former examples.

$$\text{As } \overset{\text{ft.}}{8} : \overset{\text{ft.}}{40} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{x} = 5 \text{ inches.}$$

Fig. 32.

DIAGONAL SCALE $\frac{1}{96}$.



Draw a line 5 inches long. This line represents 40 feet. Divide it into 4 equal parts; each division represents 10 feet. Number the points of division 0, 10, 20, 30, and from the extremities of the line thus divided and the points of division draw 5 perpendiculars. Subdivide the left primary division into 10 equal parts. Each subdivision represents a foot. Number the points of subdivision from right to left 0, 1, 2, 3, 4, &c. Now, as you require 12ths of a foot or inches, and not 10ths of a foot, draw 12 parallel lines, enclosing 12 equal spaces, and let these lines be about the distance apart as shown in Fig. 32, and on the extreme left perpendicular number these lines from the bottom upwards 0, 2, 4, 6, 8, 10, 12. Join the point 9 on the lowest line with 12 on the extreme left of the top line, and through the other points of division on the lowest line draw diagonals parallel to it, and the scale is completed. Any other lines drawn on the scale besides those mentioned are merely ornamental, and have nothing whatever to do with the construction of the scale.

EXAMPLE 28.

Construct a diagonal scale to show hundredths of an inch.

Draw a line 5 inches long, and divide it into 5 equal parts; each part represents an inch. Parallel to this line draw 10 equally distant parallel lines. Erect perpendiculars from the points of division on the lowest line, and on the extreme left perpendicular number the parallel lines 0, 2, 4, 6, 8, 10. Subdivide the left primary division on the lowest line into 10 equal parts; each part represents $\frac{1}{10}$th of an inch. Draw the diagonal parallels. Fig. 31, omitting the half-inch diagonally divided part, will answer for this example.

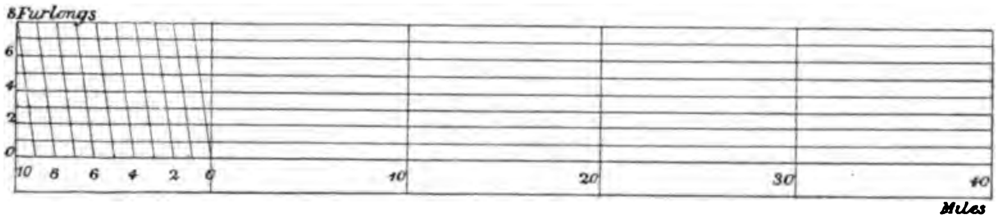
EXAMPLE 29.

Construct a scale 10 miles to an inch to measure 50 miles and show furlongs.

$$\text{As } \overset{\text{miles.}}{10} : \overset{\text{miles.}}{50} :: \overset{\text{in.}}{1} : x = 5 \text{ inches.}$$

Fig. 33.

DIAGONAL SCALE $\frac{1}{633.600}$.



Draw a line 5 inches long, and divide it into 5 equal parts; each division represents 10 miles. Erect perpendiculars from the divisional points, and subdivide the left primary division on the line into 10 equal parts; each subdivision represents 1 mile.

To get the 1-8th of one of these subdivisions, or 1 furlong, draw 8 equally distant lines parallel to the lowest line, and complete the scale as in Fig. 33.

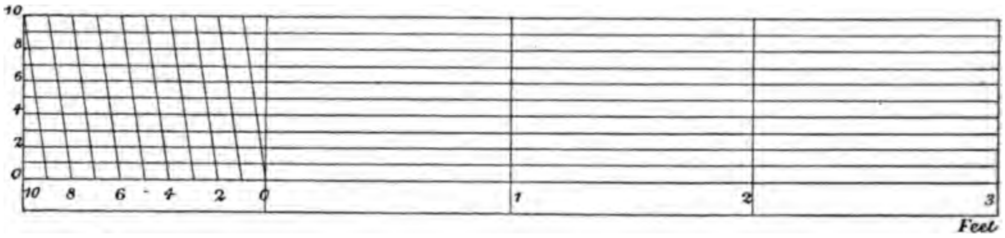
EXAMPLE 30.

Construct a scale to measure feet, tenths of feet, and hundredths of feet, when 1·25 inches represent a foot.

Here 5 inches will be a convenient length for the scale, as it contains 1·25 inches four times.

Fig. 34.

DIAGONAL SCALE $\frac{1}{9\cdot6}$.



Divide a line 5 inches long into 4 equal parts; each division is 1·25 inches long, and represents a foot. Subdivide the left primary division into 10 equal parts; each subdivision represents $\frac{1}{10}$th of a foot. To show hundredths of feet on the scale, draw 10 equally distant lines parallel to the 5-inch line thus divided, and complete the scale as in Fig. 34.

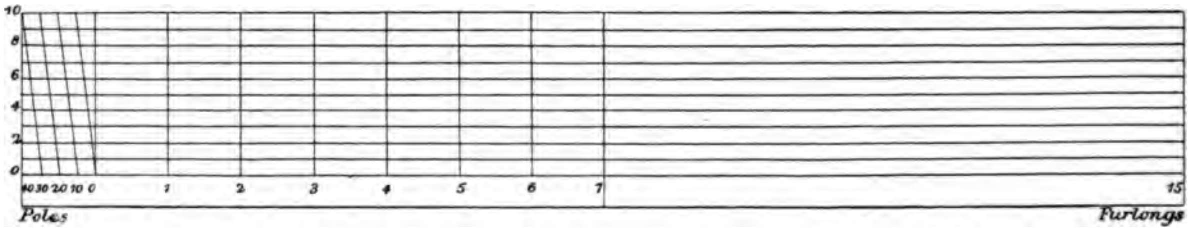
EXAMPLE 31.

Construct a scale 3 inches to a mile, to show furlongs and poles.

Here, as 3 inches represent a mile, the $\frac{1}{8}$th of 3 inches represents 1 furlong, because there are 8 furlongs in a mile; and the $\frac{1}{40}$ of a furlong will represent 1 pole or perch, because there are 40 poles in a furlong.

Fig. 35.

DIAGONAL SCALE $\frac{1}{21\cdot120}$.



Draw a line 6 inches long; this represents 2 miles. Divide it into 2 parts; each division is 3 inches, and represents 1 mile. Subdivide the left 3 inches into 8 equal parts; each subdivision represents 1 furlong. Erect perpendiculars from the points of division and draw 10 equally distant lines parallel to the 6-inch line. Again divide the left primary subdivision, which represents 1 furlong, into 4 equal parts; each part represents 10 poles. Draw the diagonal lines and number the scale as in Fig. 35.

EXAMPLE 32.

To draw a diagonal scale, the representative fraction being given.

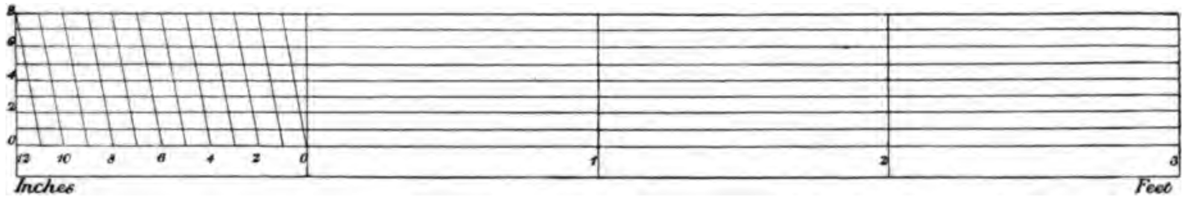
Draw a diagonal scale to measure 4 feet, and show 8ths of an inch, the representative fraction being $\frac{1}{8}$.

Here 1 inch represents 8 inches, therefore 1·5 inches represent 12 inches, or 1 foot.

$$\text{As } \overset{\text{ft.}}{1} : \overset{\text{ft.}}{4} :: \overset{\text{in.}}{1{\cdot}5} : \overset{\text{in,}}{x} = 6 \text{ inches.}$$

Fig. 36.

DIAGONAL SCALE $\frac{1}{8}$.



Draw a line 6 inches long, and divide it into 4 equal parts; each division represents 1 foot. Erect perpendiculars on the points of division, and subdivide the left primary division into 12 equal parts; each subdivision represents 1 inch.

To show 8ths of an inch, draw 8 equally distant lines parallel to the 6-inch line, and draw the diagonals.

Number the scale as in Fig. 36.

EXAMPLE 33.

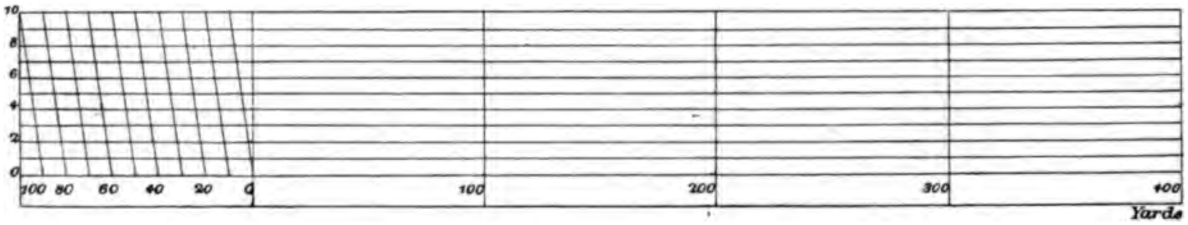
The representative fraction is $\frac{1}{3000}$; draw a diagonal scale to show yards, and measure 500 yards.

In this example 1 inch represents 3,000 inches.

$$\begin{array}{lcccccccl} & \text{in.} & & \text{in.} & & \text{in.} & & \text{in.} & \\ & & & 500 \times 36 = & & & & & \\ \text{As} & 3{,}000 & : & 18{,}000 & :: & 1 & : & x & = 6 \text{ inches.}\end{array}$$

Fig. 37.

DIAGONAL SCALE $\frac{1}{3000}$.



Draw a line 6 inches long, and divide it into 5 equal parts; each division represents 100 yards. Parallel to this line draw 10 equally distant lines, and through the divisions on the lowest line erect perpendiculars cutting the parallels. Subdivide the left primary division into 10 equal parts; each subdivision represents 10 yards. Draw the diagonals, and number the scale as in Fig. 37.

EXAMPLE 34.

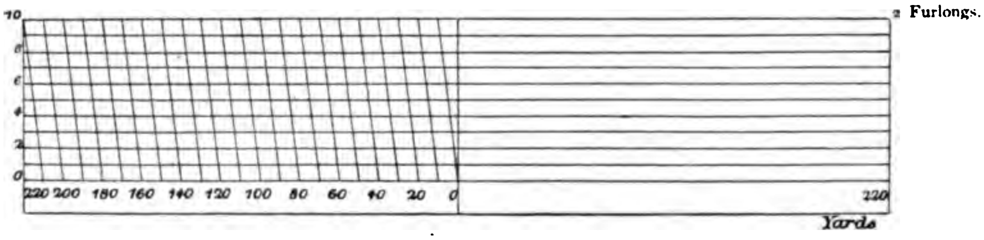
The representative fraction is $\frac{1}{3520}$.
Draw a diagonal scale to show yards.
Here 1 inch represents 3,520 inches.

$$\begin{array}{lcccccccl} & \text{in.} & & \text{in.} & & \text{in.} & & & \\ & & & 1{,}760 \times 36 = & & & & & \\ \text{As} & 3{,}520 & : & 63{,}360 & :: & 1 & : & x & = 18 \text{ inches.}\end{array}$$

The scale then is 18 inches to a mile, or $18 \div 8 = 2\frac{1}{4}$ inches to $\frac{1}{8}$th of a mile, which is 1 furlong, or 220 yards.

Fig. 38.

DIAGONAL SCALE $\frac{1}{3520}$.



Draw a line, and set off on it a number (suppose two) of distances, each 2¼ inches long. Erect perpendiculars, and parallel to the line thus divided draw 10 equally distant lines. Subdivide the left primary division, which represents 220 yards, into 22 equal parts; each subdivision represents 10 yards. Draw the diagonals, which show single yards, and number the scale as in Fig. 38.

One-fourth of a mile, or 2 furlongs, is shown in this example.

MARQUOISE'S SCALES.

These scales consist of a right-angled triangle, of which the hypothenuse or side subtending the right angle is three times the length of the shortest side, and two rectangular scales, each of which has two natural and two artificial scales cut on each side of it. Each rule is 1 foot long. The scale placed close to the edge is the artificial one, and the inner scale is the natural one. The divisions on the artificial scale are three times the length of those on the natural one, so as to have the same proportion to each other that the longest side of the triangle has to the shortest. The natural scales are simply divided scales of equal parts, the left primary divisions being subdivided into 10 equal parts, numbered from the left hand to the right.

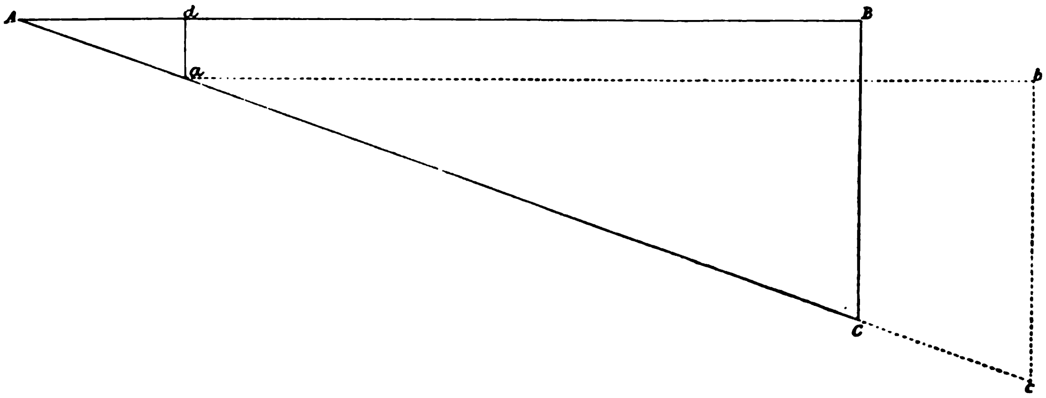
The zero point on the artificial scale is placed in the middle of the edge of the rule, and the primary divisions are numbered both ways from this point to the ends of the rule.

On one face of the scales 30 and 60 are marked, and on the other face 25 and 50 parts to an inch. On the other rules 35 and 45 and 20 and 40 parts to the inch.

The hypothenuse of the triangle has an index line drawn at right angles from the middle of it, and the longer of the other sides has a bevelled edge for convenience in drawing lines.

Lines parallel to a given line may be conveniently drawn with these scales. Suppose it is required to draw a line parallel to A B, and at a given distance from it, say half-an-inch, Fig. 39.

Fig. 39.



Place the bevelled edge of the triangle to coincide with the given line A B. Place the edge of the ruler close to the hypothenuse of the triangle. Observe the point on the scale which is marked by the index of the triangle, and holding the ruler firmly with the left hand slide the triangle along the edge of the ruler from left to right till the index has moved $1\frac{1}{2}$ inches from the starting point. A line drawn along the bevelled edge of the triangle will be parallel to A B and half an inch distant from it.

The reason of this is evident. The shortest side of the triangle being one-third the length of the longest side, the distance which it descends perpendicularly from the line A B will always be in the proportion it bears to the longest; therefore, if the longest side be moved $1\frac{1}{2}$ inches, the shortest side will have descended half-an-inch from the given parallel line.

Let A B C, Fig. 39, be the position of the triangle before it was moved, and let *a b c* be its position after being moved. Draw *a d* parallel to *b c*. The triangle A *d a* is similar to the triangle *a b c*; therefore the sides about the equal angles are proportional, that is:—

$$bc : ca :: da : a\text{A}.$$

but *b c* is $\frac{1}{3}$ of *a c*, therefore *d a* is $\frac{1}{3}$ of A *a*.

If it were required to draw a line perpendicular to a given line from a given point in it or without it. Make the shortest side of the triangle coincide with the given line and apply the edge of the rule close to the shortest of the triangle. Move the triangle along the edge of the rule till the side A B comes to the given point, through which draw a line along A B, which will be the perpendicular required.

If the triangle be accurately constructed, the Marquoise scales may with advantage be used for dividing lines accurately and expeditiously, and the military student will find them of much value in drawing the slopes and traces of fortifications.

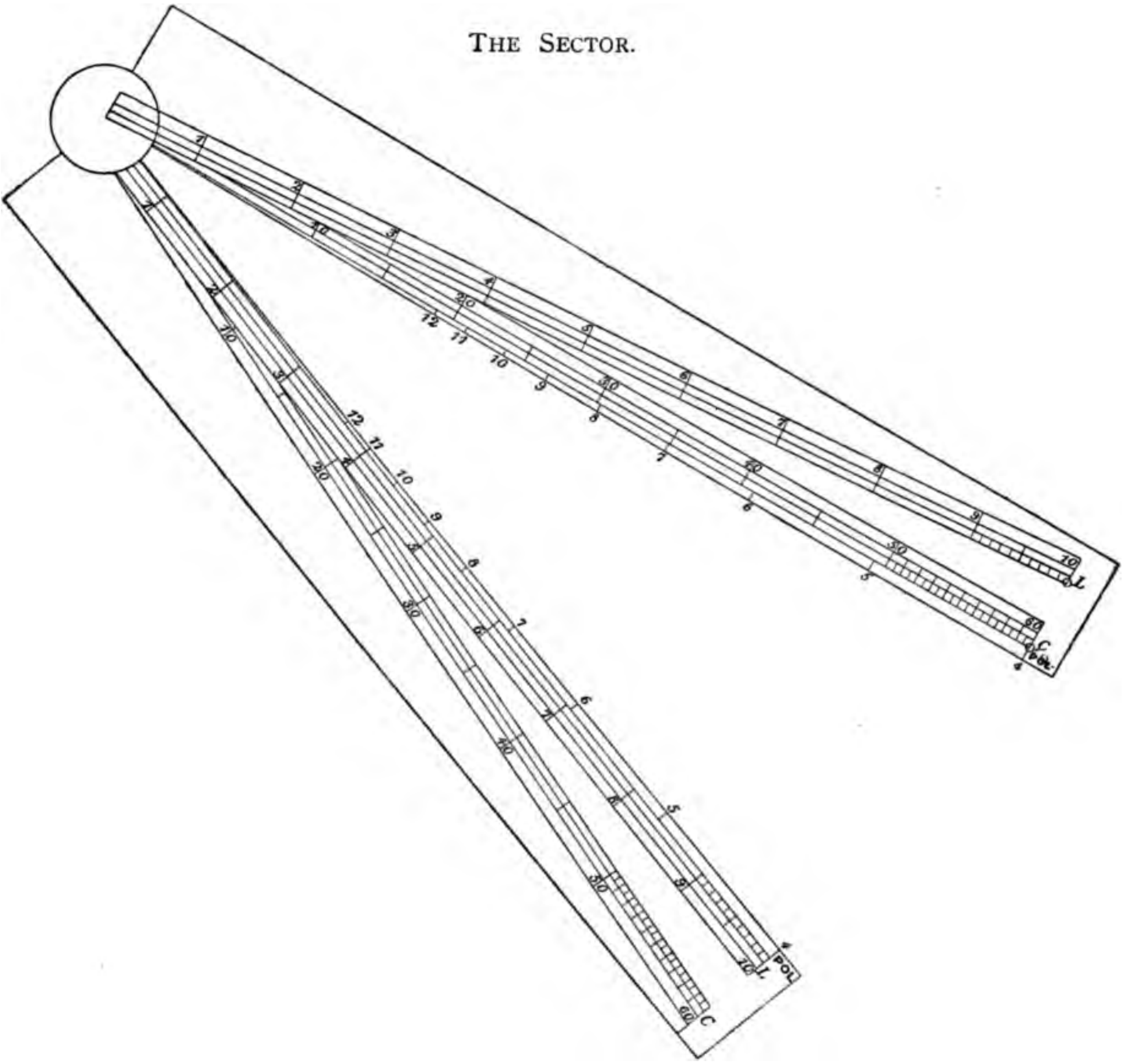
THE SECTOR.

The sector is a kind of universal scale, by the aid of which and a pair of compasses every operation in geometrical drawing may be performed. This ingenious instrument takes its name from def. 10, Euclid, 3rd Book, which defines a sector as the figure contained by two radii of a circle and the part of the circumference between them.

This instrument consists of two equal rulers, which represent the two radii, moveable about a joint, the centre of which represents the centre of the circle. Fig. 40.

Fig. 40.

THE SECTOR.



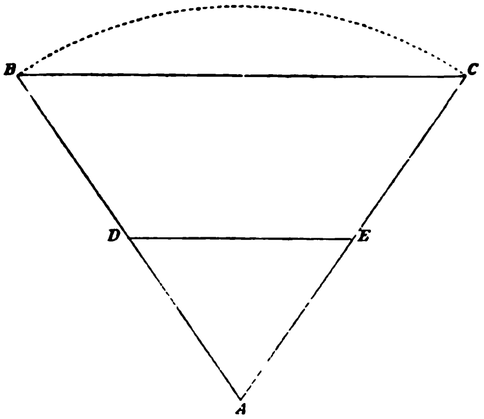
The sectoral lines are drawn in pairs from the centre, one line of each pair being on either leg of the sector. Upon one face are marked the line of lines, **L**; the line of chords, **C**; the line of secants, **S**; and the line of polygons, **POL.** Upon the other face are the lines of lines, **S**; and the lines of tangents, **T**.

The most important of these lines are the line of lines; the line of chords; and the line of polygons. These are the only sectoral lines treated of in this book, which has been drawn up chiefly for the use of military students.

Questions solved on the sector are either simple or compound. Simple when they are commenced and finished upon the same pair of lines, and compound when they are begun upon one pair and finished upon another.

When a distance is commenced from the centre, and measured along the sectoral line, it is called a lateral distance; but if it is measured from a point in one line of a pair to the corresponding point on the line of the same denomination on the other leg of the sector, it is called a transverse distance.

Fig. 41.



Let, for example, A B, A C, Fig. 41, represent two sectoral lines, and let D and E be two corresponding points on lines of the same denomination. B C and D E are transverse distances, and A D and A E are lateral distances. A B is equal to A C, being radii of the same circle, and A D and A E are made equal to one another. The triangles A B C and A D E are similar, and the sides about the equal angles are proportional; therefore A D : A B :: D E : B C; and whatever part A D is of A B, D E is the same part of B C. If A D be one-half of A B, D E is one-half of B C; and if A D be $\frac{2}{5}$ths of A B, D E is $\frac{2}{5}$ths of B C; and so on.

The application of the line of lines will be seen from the following examples:—

1. To bisect a given finite straight line.

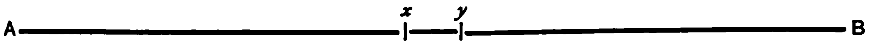
Open the legs of the sector till the transverse distance between 10 and 10 is equal to the given line; the distance between 5 and 5 will be half of the given line.

2. To divide a line into any number of equal parts, when the number of equal parts is a power of 2. Suppose 16.

Open the sector till the transverse distance between 10 and 10 is equal to the given line; then take the transverse distance between 5 and 5, and set it off from either extremity of the given line, which is thus divided into 2 equal parts.

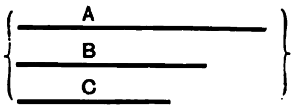
Again, make half the given line a transverse distance between 10 and 10, then the transverse distance between 5 and 5 will bisect the half-line set off between 10 and 10, thus giving one-fourth the given line; and in this manner, by continual bisection, the line can be divided into 16 equal parts.

3. To divide a line into any number of equal parts—not a power of 2. Suppose 17 equal parts. Let A B be the given line.



Make the transverse distance between 10 and 10 equal to the line A B. Take the transverse distance between 8 and 8, and set it off from A to x, and from B to y; the line will thus be divided into 3 parts, A $x = \frac{8}{17}$; B $y = \frac{8}{17}$; and $x\,y = \frac{1}{17}$ of the given line A B. By continual bisection, A x and B y may be divided each into 8 equal parts, thus dividing A B into 17.

4. To find a fourth proportional to three given lines.

Let  be the given lines.

From the centre of the sector take a point making the lateral distance equal to A. From this point make the transverse distance equal to B. Mark another point, the lateral distance of which, *from the centre*, is equal to C; then the transverse from this point will be the fourth proportional.

A third proportional to A and B may be found by making the second term a lateral distance from the end of the first term, and setting off the second term from the centre of the sector, as though it were a third term, and the fourth proportional so found will be a third proportional to A B.

THE LINE OF CHORDS.

The line of chords is used for setting off angles of any number of degrees.

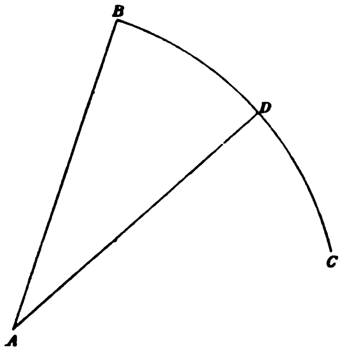
The chord of 60° in any circle is equal to the radius of that circle.

If the sector be opened till the transverse distance at 60 is equal to the chord of 60°, then the transverse distance at 50 will be the chord of 50°, the transverse distance at 30 will be the chord of 30°, the transverse distance at 25 will be the chord of 25°, and so on.

1. To make an angle of any number of degrees less than 60°. Suppose 30°.

Make the transverse distance between 60 and 60 on the line of chords equal to the radius of the circle, and with A as centre and the transverse distance as radius describe the arc B C. Take, with the dividers, the transverse distance of the given degrees 30, and make the arc B D equal to 30. Join A B, A D. The angle B A D is the angle required.

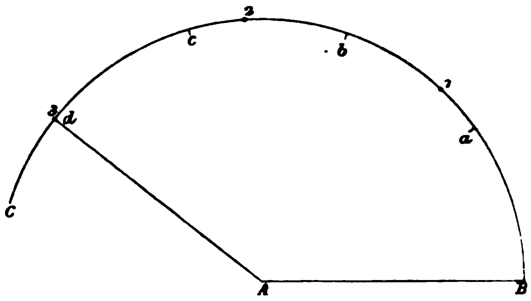
Fig. 42.



2. Make an angle greater than 60°. Suppose 144°. Fig. 43.

With the transverse distance between 60 and 60 as radius, describe the arc B C. Take one-third or one-fourth of 144°, and find the transverse distance of that number of degrees on the sector. Step off that number of degrees on the arc B C, and you have the angle required. Suppose you take one-third of 144° = 48. Take the transverse distance at 48, and set it off three times on the points 1, 2, and 3. Join 3 and B with the centre A. Then the arc 3 B, or the angle 3 A B, contains 144°. Suppose you take one-fourth of 144° = 36. Take the transverse distance at 36 and 36, and set it off from B four times in the points *a*, *b*, *c*, *d*. Join *d* A, B A, and the angle *d* A B is the angle required.

Fig. 43.



The double scales of chords upon the sector are more generally useful than the single line of chords on a plain scale or flat protractor. On the sector the arc may be of any length, but on the plain scale or protractor the arc described must be always of the same radius.

To construct a plain scale of chords.

1. Describe a quadrant of a circle A B C. B C is the chord of 90°.

2. Divide the arc B C into 9 equal parts. First trisect the arc, and then by trials divide each third part into 3 equal parts. Number the parts 10, 20, 30, &c., for each part contains the one-ninth of 90° = 10°.

3. With B as centre, and the divisions of the arc as radii, describe 8 arcs cutting the chord B C in 1, 2, 3, 4, &c. The chord B C is now a scale of chords, and each division of the scale contains 10°. Each of these divisions subdivided into 10 equal parts gives single degrees.

Fig. 44.

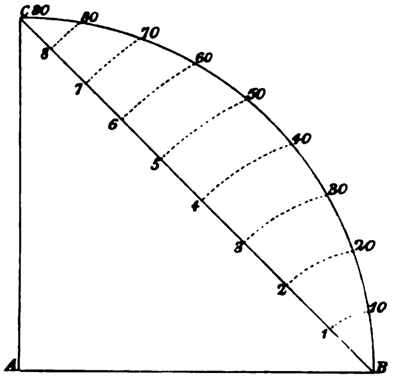
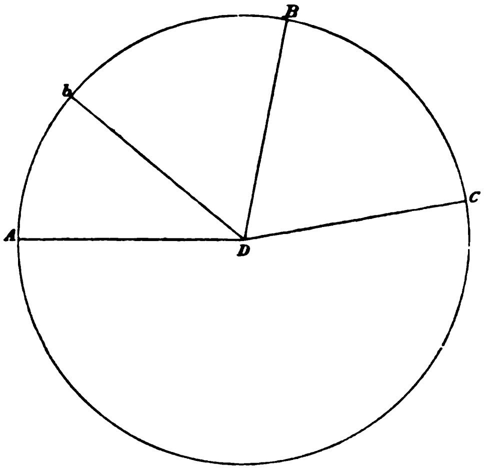


Fig. 45.



In a circle A B C, it is required to mark off angles equal to 40°, 60°, and 70°.

On the scale take the distance from 0 to 40 and mark it off from A to *b*. Mark off 60 from *b* to B, and mark off 70 from B to C. Join A *b*, B C with D. Then the angles A D *b*, *b* D B, and B D C are the angles required.

THE LINE OF POLYGONS.

The line of polygons is chiefly used for dividing the circumference of a circle into any number of parts from 4 to 12. By this means a regular polygon can be readily inscribed in a given circle.

The radius of any circle is equal to the side of a regular hexagon inscribed in that circle. If the sector be opened till the transverse distance from 6 to 6 on the line of polygons is equal to the radius of the given circle, then the transverse distance from 4 to 4 will be the side of the inscribed square; the transverse distance from 5 to 5 will be the side of a pentagon; the transverse distance from 7 to 7 will be side of a heptagon, and so on.

1. If it be required to inscribe a regular pentagon in a circle of 1·5 inches radius.

Describe the circle, and open the sector till the transverse distance between 6 and 6 is equal to 1·5 inches; then the transverse distance between 5 and 5 is the side of the pentagon.

2. If it be required to construct a regular polygon, suppose a heptagon on a given line A B.

Open the sector till the transverse distance between 7 and 7 on the line of polygons is equal to A B; then the transverse distance between 6 and 6 is the radius of a circle whose circumference can be divided by the given line A B into the required number of sides.

3. For a pentagon, open the sector till the transverse distance between 5 and 5 equals the given line, then the transverse distance between 6 and 6 is the radius of that circle; and so on, for the construction of the other regular polygons.

4. If it be required to construct, on a given straight line, an isosceles triangle having each of the angles at the base double the vertical angle.

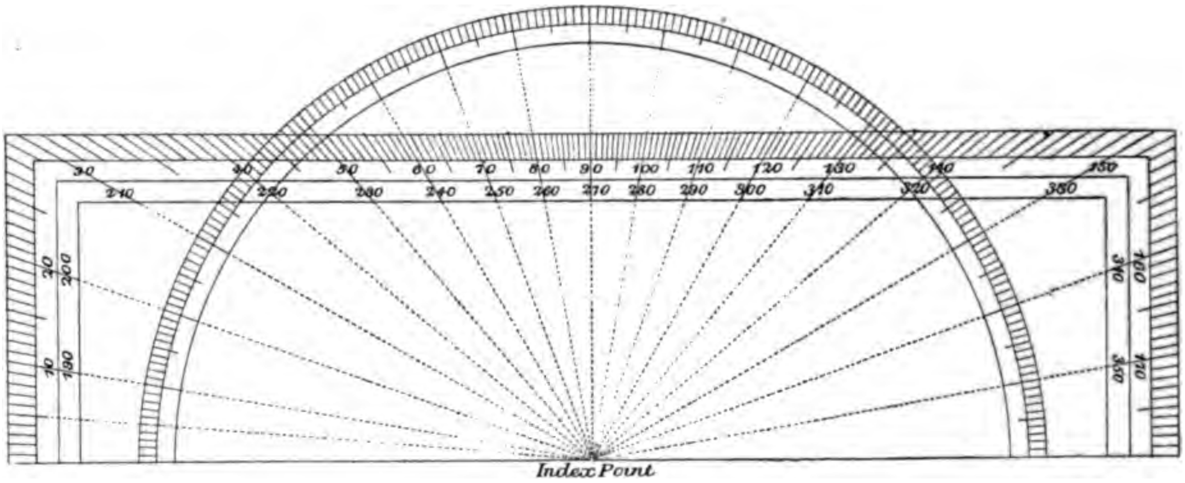
Open the sector till the transverse distance between 10 and 10 is equal to the base of the triangle; the transverse distance between 6 and 6 will be the length of each of the sides of the isosceles triangle.

THE PLAIN PROTRACTOR.

The Protractor is used for laying down angles. There are two forms of the Protractor, the semicircular and the rectangular, both of which are shown in Fig. 46.

Fig. 46.

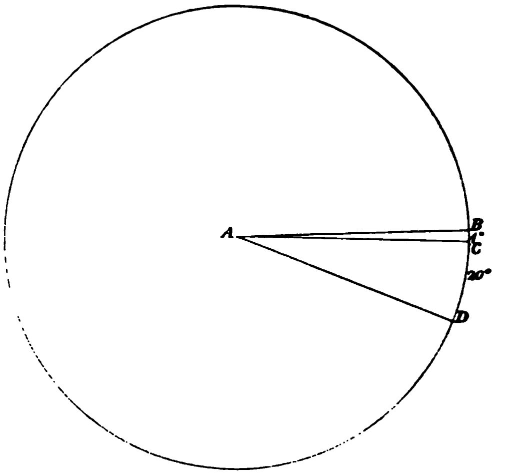
THE PLAIN PROTRACTOR.



If the circumference of a circle be divided into 360 equal parts, each part is called a **degree.** A semicircle contains 180°, and a quadrant, or fourth part of the circumference, contains 90°.

The division of the semicircle into 180° shows the principle on which the protractor is constructed. Single degrees are cut on three edges of the rectangular protractor, and are numbered in tens. The index point on the plain edge is that from which angles are set off.

Fig. 47.

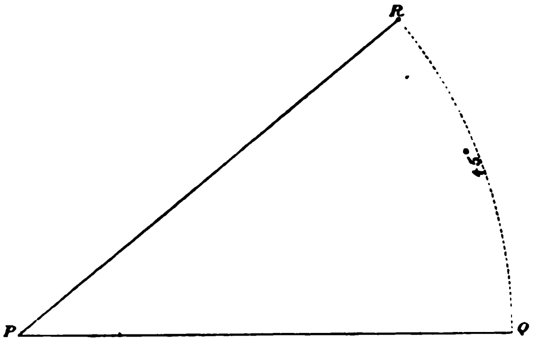


Angles are computed by the number of degrees they contain. Suppose that in the circle, Fig. 47, the distance from B to C is $\frac{1}{360}$ part of the circumference, then the angle B A C is an angle of 1°. If the part of the circumference between C and D contain twenty such parts, then the angle C A D is an angle of 20°, and so on.

Fig. 48.

If it be required to set off an angle of 45° from the point P in the line P Q. Fig. 48.

Make the edge of the protractor coincide with the line P Q, so that the index point on the base of the protractor coincides with the point P. Then make a dot on the paper where the line on the upper edge of the protractor defines 45°. Call this point R. Remove the protractor and join R P. Then the angle R P Q is an angle of 45°, and in a similar manner any angle may be set off.



THE MILITARY PROTRACTOR.

This instrument differs from the ordinary protractor in having a number of parallel lines drawn across it at right angles to its long edge. It is used for laying down bearings which have been taken with the prismatic compass or other similar instrument. The plain edge of the ruler on which the zero point is marked may represent the magnetic meridian from north to south, and the parallel lines drawn at right angles to it across the ruler may represent magnetic east and west lines. These parallel lines are drawn at such distances from each other as will form a scale of equal parts. The scale may be 12 inches, 6 inches, or 4 inches to a mile. Such scales are convenient for laying down military sketches.

The paper on which the military drawing is to be made has a number of parallel lines ruled on it at unequal distances from each other. These lines also represent magnetic lines from east to west.

Suppose you want to lay down the following bearings, taken from a fixed point A, with the prismatic compass. The bearing to A *a*, N. 30° E., to A *b*, N. 120° E., to A *c*, N. 145° E., to A *d*, N. 45° W.

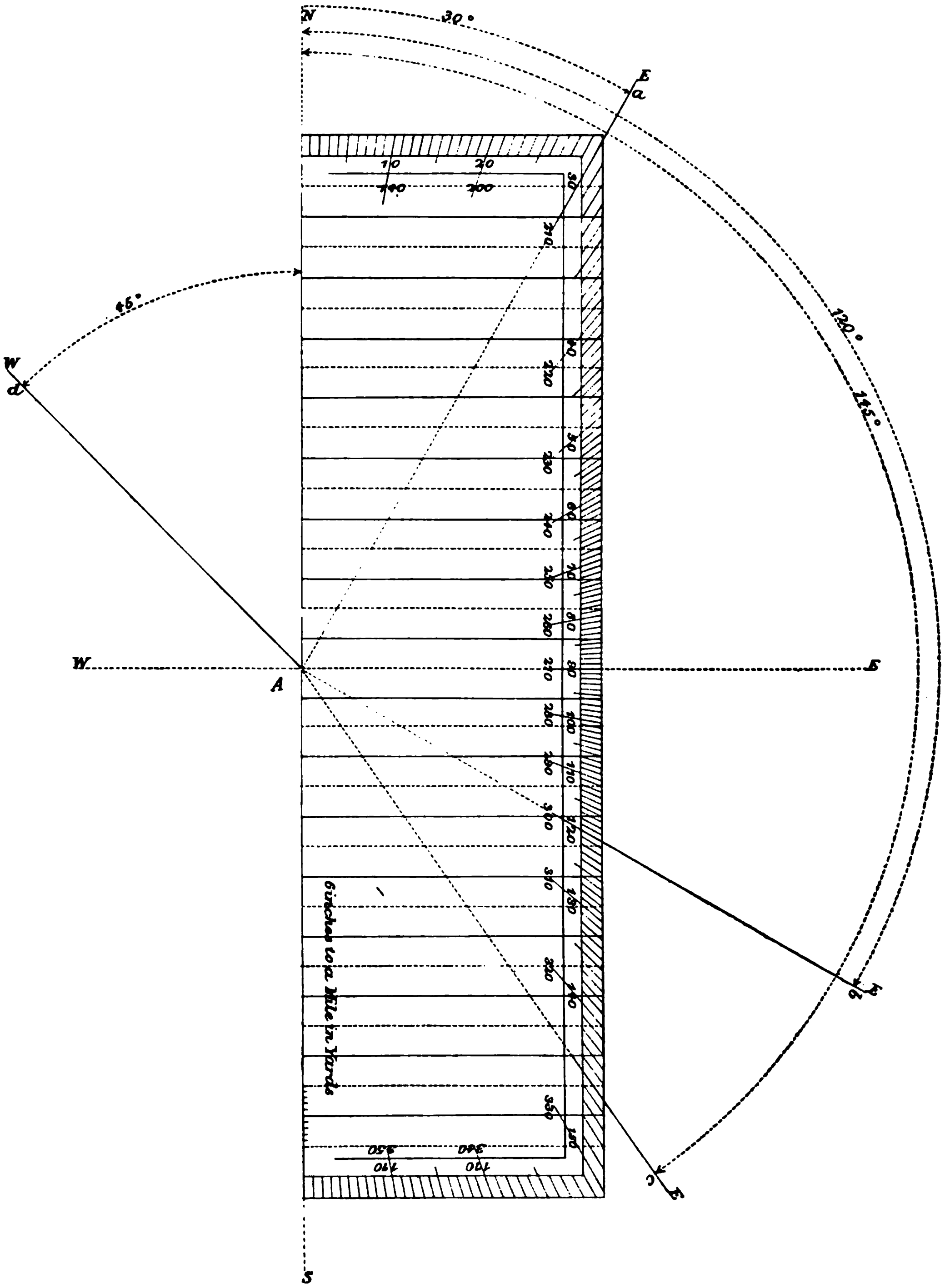


Fig. 49.

MILITARY PROTRACTOR.

Mark on any convenient part of the paper on which the drawing is to be made a point A, to represent the point from which the bearings are taken. Place the protractor on the paper with the zero point at A, as in Fig. 49, and adjust the instrument so that one of the parallel lines upon it may coincide with one of the parallel lines on the paper. Suppose the line from the index or zero point on the protractor coincides with the line W E on the paper. The edge of the protractor now represents the magnetic meridian from north to south. From the north point of the protractor mark off the angle to *a*, N. 30° E., the angle to *b*, N. 120° E., the angle to *c*, N. 145° E. To mark off the angle to *d*, keep the protractor still in the same position, and prick off from the south end of the protractor an angle S. 45° E. Draw a line from the point pricked off to A, and produce it to *d*, which will give the bearing required; for the angle which points S. 45° E. points N. 45° W., the vertical angles being equal.

Remove the protractor, and join A*a*, A*b*, A*c*, A*d*, which give the bearings required.

Suppose bearings 270° and 315° are taken with the prismatic compass, and you wish to lay them down with the military protractor. Select a convenient point on your paper, and as the bearings to be laid down are more than 180°, place the index point of the protractor on the point selected, having the graduated edge of the protractor on the left of *that* point, and not on the right of it, as in Fig 49. Make any of the parallel east and west lines on the protractor coincide with one of the parallel lines ruled on the paper, and mark off the required degrees, which are numbered on the inner line on the protractor. The dotted line A *w*, Fig. 49, bears 270° from the point A, and A *d* bears 315° from the same point.

The parallel lines on the protractor have nothing to do with the division of the protractor into degrees. The student will find them ruled closer on one protractor than on another. They simply represent a scale of yards. If closely ruled, they may represent a scale 4 inches to a mile in yards. In Fig. 49 they show a scale 6 inches to a mile in yards. On ivory protractors these parallel lines are drawn alternately in black and red for convenience in laying down distances. The dotted parallel lines on the protractor, Fig 49. may represent the red lines on a similar protractor on ivory; and the distance between two dotted lines or two plain lines represents 100 yards.

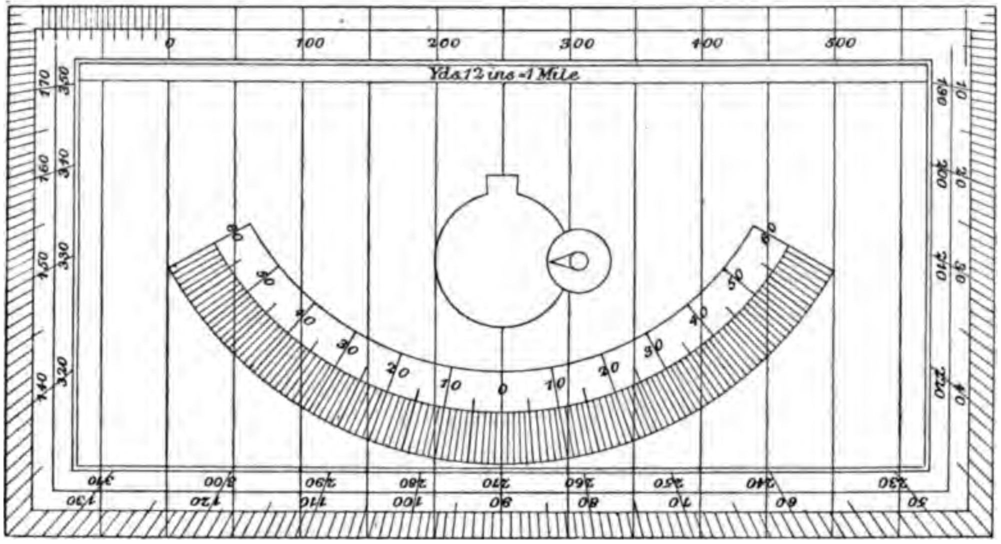
THE CLINOMETER AND OROMETER.

The Sandhurst Military Protractor has a graduated arc marked on one side of it, which, with a plummet and string, is used for finding the different degrees of the slopes of hills. Fig. 50. This arc with the plummet and string is called a Clinometer; and the protractor with the tables and scales at the back is called an Orometer. On the reverse side of this protractor is a table showing the approximate horizontal distances in yards of contours for different degrees of slope on 12-inch and 6-inch scales. A scale of shade for the different degrees of slope of 25 feet contours on a 6-inch scale; and a column of horizontal equivalents for 25 feet contours. Fig. 50.

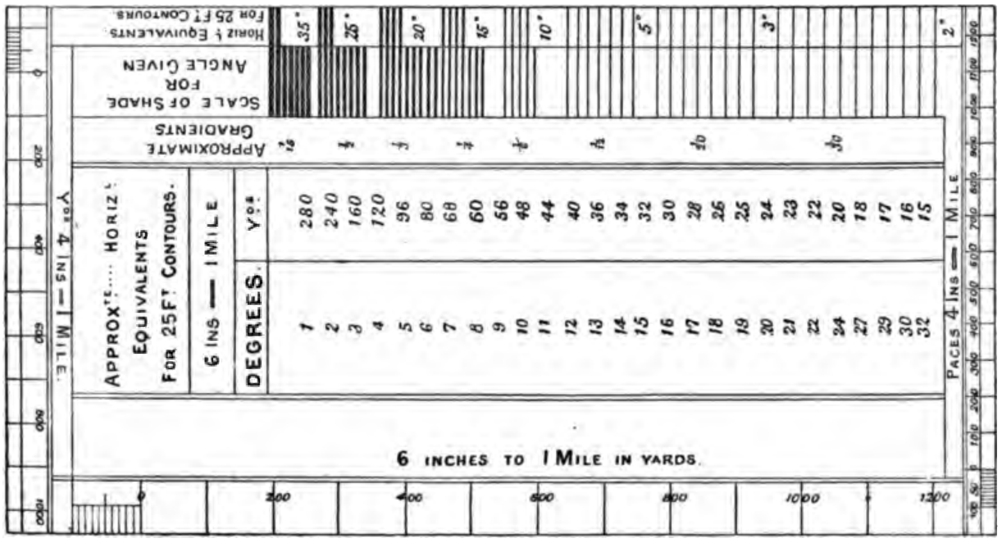
SANDHURST PROTRACTOR OR OROMETER.

Fig. 50.

FRONT.



BACK.


Horiz¹ Equivalents For 25 Ft Contours.
35°
25°
20°
15°
10°
5°
3°
2°
Scale of Shade for Angle Given
Approximate Gradients
Approx^te Horiz^l Equivalents for 25 F^t Contours.
6 Ins — 1 Mile
Y^ds
Degrees
Y^ds 4 Ins — 1 Mile.
Paces 4 Ins — 1 Mile
6 inches to 1 Mile in yards.
0
200
400
600
800
1000
1200

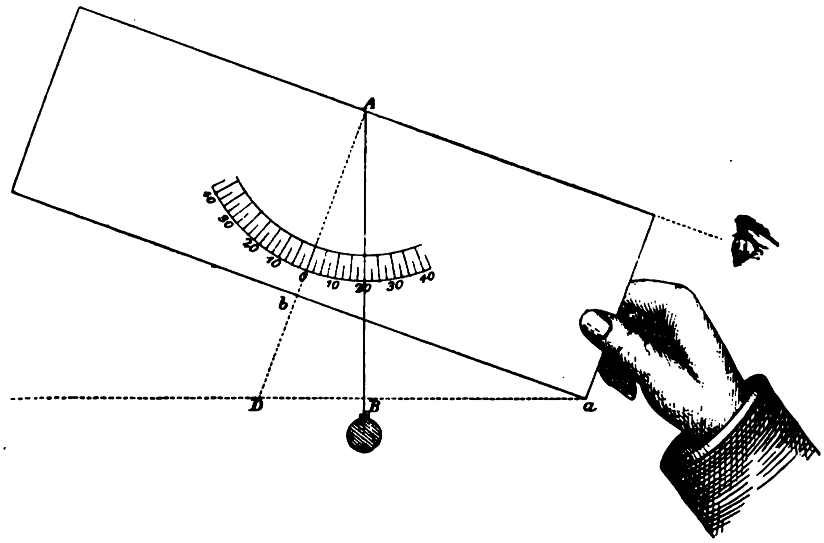
DEGREES.	Y^ds
1	280
2	240
3	160
4	120
5	96
6	80
7	68
8	60
9	56
10	48
11	44
12	40
13	36
14	34
15	32
16	30
17	28
18	26
19	25
20	24
21	23
22	22
24	20
27	18
29	17
30	16
32	15

Around the edges or on any other convenient part of the protractor, scales are drawn showing respectively 12, 6, or 4 inches to a mile in yards, and a scale of 8 or 4 inches to a mile in paces.

The parallel lines representing east and west lines are drawn across the protractor representing a scale of 12, 6, or 4 inches to a mile in yards.

Fig. 51.

THE CLINOMETER.

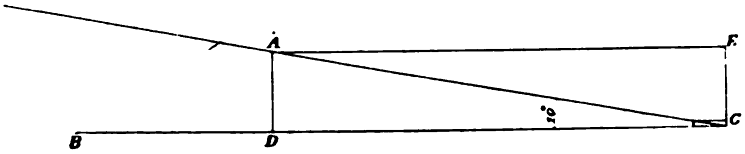


To find the degree of slope in a hill, hold the protractor to the eye, and incline it till you sight, along the upper edge, the surface of the hill. Fig. 51. The line of the plummet, hanging vertically, will mark on the arc the degree of slope ; the angles D A B and D *a b* being equal.

Now having found the degree of slope, by reference to the table, the approximate horizontal distance will be found in its proper column, according as it may be required for a 12 or 6-inch scale. This horizontal distance will represent the base of a right-angle triangle, whose hypothenuse represents the slope of the hill, and whose perpendicular is 25 feet on a 12 or 6-inch scale.

Suppose we select a 6-inch scale, and that the angle of inclination taken with the graduated arc of the protractor is 10°.

Fig. 52.



Draw the line B C of any reasonable length. From C set off the angle A C D equal to 10°. On C erect C E perpendicular to C D, and equal to 25 feet. Draw A E parallel to the horizontal line B C. From A draw A D perpendicular to D C; D C is the approximate horizontal distance. If the distance D C be taken between the points of the compass and applied to the scale of yards (6 inches to a mile), it will be found to measure 48 yards. The ratio of the altitude of A D to D C would be about $\frac{1}{8}$. In order to make the illustration plain, the height A D is exaggerated twelve times, and D C is exaggerated accordingly.

EXERCISES ON SCALES.

In each of the following exercises, which are taken from examination papers, the scales should be properly figured, the calculations shown, and the representative fraction given.

1. Draw a plain scale of miles and furlongs long enough to measure 6 miles, in which $2\frac{1}{2}$ miles is represented by 2·35 inches.

2. Construct a diagonal scale to measure feet, inches, and eighths of an inch, on a scale of $\frac{7}{8}$ths of a foot to an inch.

3. On a given plan a distance of $\frac{3}{4}$ths of a mile measures $4\frac{1}{2}$ inches. Construct a scale of yards for this plan to measure distances from 50 up to 1,000 yards.

4. Draw a plain scale of 6 inches to a mile, showing furlongs; and a comparative scale of yards by which distances of 40 yards may be measured.

5. Construct a diagonal scale of feet and inches, on which 17 feet would be represented by 1·6 inches. Show 50 feet.

6. Draw a scale 15 yards to an inch, showing exactly 50 yards.

7. The length of an ordinary pace is 30 inches, in stepping short it is 21 inches. Draw a scale of $\frac{1}{3000}$ to show 600 ordinary paces, and also a comparative scale of short paces.

8. Draw a plain scale of 665 paces to an inch, 100 paces being the least and 5,000 paces the largest dimension shown. Assume a pace to measure 32 inches.

9. A scale of 60 Russian versts measure 7·5 inches. Draw a comparative scale of English miles showing furlongs diagonally.

10. On a plan, $7\frac{1}{2}$ miles are represented by $1\frac{1}{4}$ inches. Construct a plain scale showing 40 miles. To the same scale apply the diagonal method to show furlongs.

11. Construct a diagonal scale of chains and links for a plan on which 27 chains are represented by 31 inches. Show 5 chains.

12. Draw a plain scale of feet and inches to measure 7 feet, $1\frac{1}{2}$ feet being represented by 1·3 inches. Draw also a diagonal scale comparative to the above, to show mètres, tenths of mètres, and hundredths of mètres.

13. A line of $1\frac{5}{8}$ inches long represents 5 feet. Construct a scale to show feet.

14. Construct a scale on which 80 feet are represented by 6 inches. Show 50 feet.

15. Draw a diagonal scale of 120 feet to an inch, to measure single feet. Show 700 feet.

16. A plan is drawn 88 feet to an inch. Draw a comparative scale by which any distances from 5 yards to 150 yards may be measured.

17. Construct a diagonal scale of 45 feet to an inch, by which single feet may be measured. Show 250 feet.

18. Construct a scale of $\frac{1}{3750}$ to show 1,000 paces of 30 inches each.

19. Draw a diagonal scale to measure miles, furlongs, and chains, 50 furlongs being represented by 9 inches.

20. What is the representative fraction of a scale in which 210 yards are represented by 1·75 inches?

21. Draw a diagonal scale to measure perches, yards, and feet, 30 yards being represented by 7 inches.

22. Construct a scale of feet to measure all distances between 70 feet and 1 foot, 5·5 feet being represented by ·52 inches. Apply the diagonal method to this scale to show inches.

23. Construct a plain scale in which 57 feet are represented by 7·2 inches, 50 feet being the greatest and 1 foot the least dimension shown.

24. Draw a scale of $\frac{1}{100}$ to show chains and links.

25. The relative areas of a map, and the country it represents, are 121 square inches, and 144 square miles. Draw the scale of the map to show miles and furlongs, and (diagonally) chains.

26. Two buoys in a harbour are distant from each other 15 kilomètres; on a drawing of the harbour their plans are 8·5 inches apart. Draw a scale of English miles for the plan.

27. Draw a plain scale of $\frac{1}{8}$th of a mile to an inch, to measure feet up to 4,000, the least dimension shown being 100 feet.

28. Draw a diagonal scale of 70 paces to the inch to measure all distances between 1 and 500 paces. From the scale draw a line 253 paces long. Assume the pace to measure 33 inches.

30. On a given plan 13 feet are represented by $1\frac{3}{8}$ inches. Construct a plain scale of feet for the plan so as to show 50 feet.

To the same scale apply the diagonal method to show inches.

31. A length of 8 feet 9 inches measures on a plan $2\frac{3}{4}$ inches. Draw a diagonal scale of feet and inches to suit the plan. Show 20 feet.

32. A distance of 730 yards is represented on a map by 4·25 inches. Draw a plain scale of yards for the map by which distances of 25 yards may be measured. Show 1,000 yards. Draw a comparative scale of chains showing 40 chains.

33. Draw a plain scale to show mètres and tenths of mètres, 5·3 mètres being represented by 7·4 inches. Draw a diagonal scale comparative to the above to show yards, feet, and (diagonally) inches.

From the diagonal scale draw a line 2 yards, 2 feet, 2 inches long. Assume a mètre = 39 inches.

34. I have a French plan in which the scale has been omitted. One of the lines on the plan, however, is numbered 804·584 mètres, and measures exactly 2.5 English inches. Construct an equivalent English scale to read 10 yards. Assume a yard = 0·9143 mètres.

35. Construct a scale of feet, inches, and sixteenths of an inch on the diagonal principle. The foot to be made = $\frac{5}{8}$ths of a inch

36. The representative fraction is $\frac{1}{102}$. Draw a plain scale to show 51 feet. Explain the principle of the diagonal scale.

37. Construct a scale to measure 78 feet, the representative fraction being $\frac{1}{168}$; and show the method of dividing such a scale geometrically.

38. The length of a plan is 4 miles. Construct a scale to show hundredths and thousandths of yards.

39. Draw a scale 2 chains to an inch to measure 264 yards.

40. A plan is drawn to a scale of 4 inches to a mile. Draw a diagonal scale to suit the plan, showing tens, fifties, hundredths, and thousandths of yards.

41. Construct a scale, in which 5 furlongs are represented by ·9 inches, to show miles and furlongs, and (diagonally) chains. The scale should measure 5 miles.

42. On a map, 5·5 feet are represented by ·52 inches. Draw a scale of feet showing inches diagonally. The scale should be long enough to measure 70 feet.

43. Construct a diagonal scale, the representative fraction of which is $\frac{1}{500}$, to measure mètres, décimètres, and centimètres.

44. Find the representative fraction of a scale in which 210 yards are represented by 1·75. Construct a scale of 13 yards to 1 inch, and construct a diagonal scale of $\frac{1}{570240}$ reading furlongs.

45. Construct a scale, the representative fraction of which is $\frac{1}{844800}$, to show 80 miles. Construct a comparative scale showing 100 versts.

46. Draw a diagonal scale to measure perches, yards, and feet. Thirty yards are represented by 7 inches. From this scale draw a line 3 perches, 2 yards, and 1 foot in length.

47. Draw a diagonal scale of furlongs and chains to measure 50 furlongs. Twenty-three furlongs are represented by 2·7 inches.

48. The representative fractions of a scale is $\frac{1}{10560}$. Construct a scale of chains to show 1 mile., The smallest unit 1 chain.

49. A map is drawn to a scale of $\frac{1}{25440}$. The map is to be increased in size to a scale 8 feet to an inch. Draw both scales.

50. Draw a scale of feet to measure 500 feet, the least dimension being 10 feet. Sixty feet are represented by ·75 inches.